

## Multi-objective Rao algorithm in resilience-based optimal design of water distribution networks

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### ABSTRACT

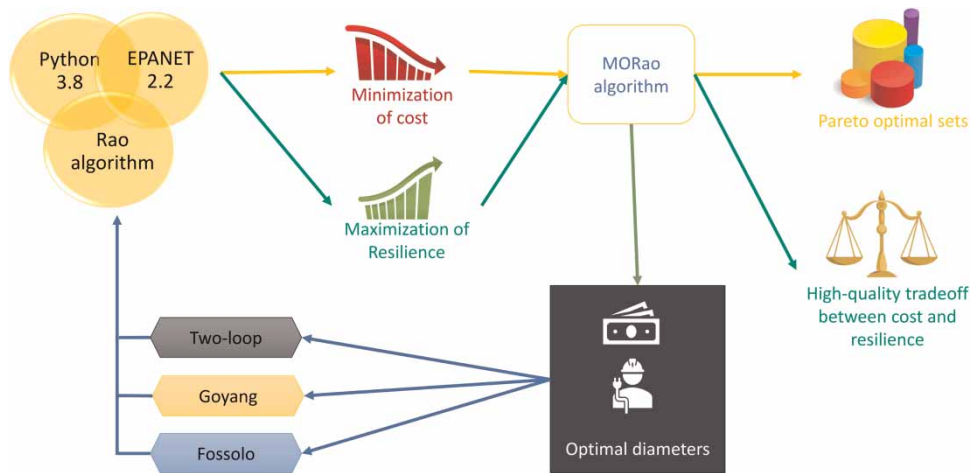
Multi-objective meta-heuristics are used to optimize water distribution networks (WDNs) as they can achieve a near-optimal balance between cost and resilience in a unified platform. Most of these algorithms include tuning of algorithm-specific control parameters for higher optimization efficiency, leading to an increased computational effort. The current study is inspired by the desire to address the above problem. The goal is to formulate a multi-objective Rao algorithm (MORao) considering an existing modified resilience index (MRI) in the optimal design of WDNs. The model is demonstrated to attain Pareto-optimal solutions to complex WDN problems without exclusive parameter tuning. The algorithm is written in Python and is linked to a hydraulic model of a WDN implemented in EPANET 2.2 using pressure-driven demand (PDD) analysis. The method is demonstrated on three widely used networks: Two-loop, Goyang, and Fossolo. The Pareto-optimal solutions examine a tradeoff between two objectives to recognize competitive solutions. The network's resilience is increased 2.5 times by only 0.8 times increase in least-cost of Two-loop network. This research indicates that this method can achieve a satisfactory level of performance with a limited number of function evaluations.

**Key words:** evolutionary algorithms, multi-objective optimization, pressure-driven demand analysis, Rao algorithm, resilience, water distribution network

### HIGHLIGHTS

- A novel parameter-less Rao algorithm is used in resilience-based optimization of water distribution networks for the first time.
- Scalarized objective function is formulated using weights and normalization.
- A practical and straightforward approach is proposed and evaluated for the complex design problem.
- Approximate Pareto curves are achieved.
- Most beneficial alternatives from Pareto optimal sets are selected.

# GRAPHICAL ABSTRACT



# ABBREVIATIONS

WDN	Water distribution network
MORao	Multi-objective Rao algorithm
DD	Demand-driven
PDD	Pressure-driven demand
TLN	Two-loop network
GYN	Goyang network
FOS	Fossolo network
EA	Evolutionary algorithm
NFE	Number of function evaluation
PF	Pareto front
$Q_{j,avl}$	Available demand at node j in $m^3/s$
$Q_{j,req}$	Required demand at node j in $m^3/s$
$H_{j,avl}$	Available head at node j in metres
$H_{j,req}$	Required/nominal head at node j in metres
$H_{j,min}$	Minimum head at node j in metres
$H_{j,max}$	Maximum allowable head at node j in metres
C	Network cost
$D_i$	Diameter of pipe i in metres
$C(D_i)$	Cost of pipe per unit length corresponding to diameter D for pipe i
$L_i$	Length of pipe i in metres
$C_{min}$	Minimum possible cost of network with given pipe sizes
$C_{max}$	Maximum possible cost of network with given pipe sizes
MRI	Modified resilience index
$MRI_{min}$	MRI corresponding to $C_{min}$
$MRI_{max}$	MRI corresponding to $C_{max}$
w	Weight
$H_f$	Head-loss due to friction in metres
$C_{hw}$	Hazen-Williams roughness coefficient
ns	Number of available pipe sizes
nn	Number of nodes
np	Number of pipes
npl	Number of pipes in a loop
$V_{k,avl}$	Available velocity at pipe k in m/s
$V_{k,min}$	Minimum required velocity at pipe k in m/s
$V_{k,max}$	Maximum allowable velocity at pipe k in m/s
p	Penalty multiplier
$\lambda$	Penalty coefficient
$r_1, r_2$	Random numbers in range (0, 1)

$D_{\text{new},i}$	Diameter for the new population for pipe $i$
$D_{\text{old},i}$	Diameter conforming to old population for pipe $i$
$D_{\text{best},i}$	Diameter conforming to best solution for pipe $i$
$D_{\text{worst},i}$	Diameter conforming to worst solution for pipe $i$
$D_{\text{rand},i}$	Diameter conforming to random solution for pipe $i$

## INTRODUCTION

An essential part of urban utilities, water distribution networks (WDNs) should be designed and operated economically and reliably. A good WDN should meet the demand at each node while maintaining the required pressure. WDN problems are classified as highly complex in terms of computing. Hence, developing a simple and effective algorithm to resolve such problems can significantly benefit decision-makers, mainly due to various constraints and multi-objectiveness.

Much work has been done in the past two decades to find the tradeoff between cost and network efficiency, such as mechanical and hydraulic reliability. Reliability can be described as the capacity to provide sufficient water supply in both normal and abnormal conditions (Farmani *et al.* 2005). Resilience is directly linked to the concept of reliability; a network cannot be resilient if it is not first reliable. Resilience is the ability to withstand extreme events. Since cost and reliability are mutually exclusive parameters, improving one necessitates compromising the other.

Even though there are various reliability estimation procedures, most of them are computationally costly. Reliability surrogate measures (RSM) are comparatively inexpensive to evaluate a desirable property, excess power, or redundancy of the WDN to quantify how resilient the device can be under stressed conditions.

Different indicators for network reliability have been introduced in past studies. Initially, Todini (2000) developed a resilience index (RI), which depends upon the excess power available to work in abnormal conditions. RI is the most commonly used second objective in WDN optimization (Farmani *et al.* 2005; Suribabu 2017). Statistical flow entropy (S), along with the modifications such as cross-entropy (CE) and entropy-based reliability index (ERI), is used to enhance the mechanical reliability of the network (Tanyimboh & Templeman 2000; Atkinson *et al.* 2014). Prasad & Park (2004) combined redundancy in loop diameters with Todini's RI as a competent reliability measure called network resilience index (NRI). NRI is widely adopted by researchers, such as Vasan & Simonovic (2010), Wang *et al.* (2015) and Bi *et al.* (2016). Jayaram & Srinivasan (2008) developed a modified resilience index (MRI), which solved the drawback of RI and NRI in dealing with multiple water sources. Numerous comparisons (Baños *et al.* 2011; Creaco *et al.* 2015; Monsef *et al.* 2019; Wang *et al.* 2019) were performed to make it easy for the researchers to select an efficient RSM. Creaco *et al.* (2015) and Wang *et al.* (2019) claimed that the resilience-based surrogate indicators (RI, NRI, and MRI), based solely on nodal pressures, are strongly correlated to network reliability in comparison to entropy-based indicators (S and CE). However, the results are not conclusive since some of the RSMs seem to be correlated with either mechanical reliability, hydraulic reliability or a combination of the two (Paez & Filion 2019). For the benefit of researchers, a comprehensive review of the application of multi-objective meta-heuristics and hyper-heuristics, and the adopted reliability surrogate measures in the optimal design of WDNs is shown in Table 1, along with the illustrated benchmark WDNs.

Researchers have used a wide variety of multi-objective evolutionary algorithms (EAs) along with their hybridization as they can efficiently approximate the Pareto front (PF; Tanyimboh & Seyoum 2020). Most of these algorithms involve tuning algorithm-specific control parameters along with standard controlling parameters (population size and number of generations), which is crucial for better efficiency. Mainly, multi-objective algorithms such as commonly used non-dominated sorting genetic algorithm-II (NSGA-II) and strength Pareto evolutionary algorithm-II (SPEA-II) require tuning of mutation rate, crossover probability, tournament size, and mutation step size. Further, the use of hyper-heuristic algorithms such as AMALGAM and Borg leads to an increase in the number of parameters such as distribution indices and probabilities of crossover and mutation, scaling factors and so on (Wang *et al.* 2015), all of which require more computational effort to tune.

A new technique based on parameter-less Rao algorithm has been applied to optimize WDNs that eliminates algorithm-specific control parameters. Recently developed by Rao (2020), Rao algorithm is formulated for the multi-objective optimization, called multi-objective Rao (MORao) algorithm. This approach, like other algorithms, necessitates modifying the penalty function and the number of function evaluations based on the network size. The proposed method demonstrated three widely used WDNs. The effect of number of function evaluations for all network cases is shown using convergence plots.

Also, computing performance may be a challenge if EAs are used for complicated issues with multiple conflicting goals (Bi Dandy & Maier 2016). In this regard, the main task of decision-makers is to satisfy the utility of the beneficiaries. Available

**Table 1** | Literature survey of various studies based on evolutionary algorithms for the multi-objective optimization of water distribution networks

Reference	Reliability measure	Algorithm	Benchmark networks
Tanyimboh & Templeman (2000)	S	NLP	<sup>a</sup>
Todini (2000)	RI	SHT	TLN
Prasad & Park (2004)	NRI	GA	TLN, HAN
Farmani <i>et al.</i> (2005)	RI, MSH	NSGA-II	AT
Jayaram & Srinivasan (2008)	MRI	Combined NSGA-II with HM	<sup>a</sup>
Vasan & Simonovic (2010)	NRI	DE	HAN, NYT
Baños <i>et al.</i> (2011)	RI, NRI, MRI	SPEA-II	TLN, HAN
Atkinson <i>et al.</i> (2014)	RI, ERI, MSH	NSGA-II	AT
Wang <i>et al.</i> (2015)	NRI	NSGA-II, $\epsilon$ -MOEA, $\epsilon$ -NSGA-II, AMALGAM, Borg	TRP, TLN, BAK, NYT, BLA, HAN, GYN, FOS, PES, MOD, BIN, EXN
Liu <i>et al.</i> (2017)	API, PHRI	$\epsilon$ -NSGA-II	HAN, FOS
Suribabu (2017)	RI, MRI	DE	HAN
Paez & Filion (2019)	MRE, HRE	NSGA-II	HAN, FOS
Nyahora <i>et al.</i> (2020)	UC + RI	NSGA-II	HAN
Sirsant & Reddy (2020)	CERI, CENRI	NSGA-II	TLN, GYN, FOS

Note: S, statistical flow entropy; RI, resilience index; NRI, network resilience index; MSH, minimum surplus head; MRI, modified resilience index; ERI, entropy-based resilience index; API, available power index; PHRI, pipe hydraulic resilience index; MRE, mechanical reliability estimator; HRE, hydraulic reliability estimator; UC, uniformity coefficient; CERI, combined entropy and resilience index; CENRI, combined entropy and network resilience index; NLP, non-linear programming; SHT, simple heuristic technique; GA, genetic algorithm; NSGA-II, non-dominated sorting genetic algorithm II; HM, Heuristic method; SPEA-II, strength pareto evolutionary algorithm II; DE, differential evolution;  $\epsilon$ -MOEA, elitist multi-objective evolutionary algorithm;  $\epsilon$ -NSGA-II, elitist NSGA-II; TLN, Two-loop; HAN, Hanoi; AT, Anytown; NYT, New York Tunnel; TRP, two reservoir problem; BAK, BakRyan; BLA, Blacksburg; GYN, Goyang; FOS, Fossolo; PES, Pescara; MOD, Modena; BIN, Balerna irrigation; EXN, Exeter network.

<sup>a</sup>Illustrated networks are not benchmark problems.

methods either design a WDN by considering a unique objective or yield a set of nondominated alternatives. In this work, the optimal design alternatives of WDN are determined in a unified platform by considering two objectives: (1) minimization of cost (for investors); and (2) maximization of MRI (for consumers). The scalarizing technique and normalized objective functions achieve weightage-based resilient design using PDD analysis. Apart from searching for a single solution with the global best results, this methodology aims to find a set of diverse solutions that define a balanced tradeoff between both objective functions, within the computational budget. Thus, the approach allows decision-makers to select final results as per the weightage importance of cost and resilience. Furthermore, a Nash bargaining model was used to identify competitive solutions from the Pareto optimal set.

## MULTI-OBJECTIVE OPTIMIZATION MODEL

The MORao algorithm written in Python code is linked to a WDN model constructed in EPANET 2.2 for the hydraulic analysis (Rossman 2000) using pressure-driven demand (PDD) analysis.

### PDD analysis

The main aim of the hydraulic analysis is to analyze the pressure at nodes and flow velocity in pipes such that it satisfies the demand at each node with adequate pressure. Higher demand uncertainty poses complexity in design and analysis of WDNs. A hydraulically reliable design should accommodate inevitable demand variations (Mangalekar & Gumaste 2021). The conventional method of hydraulic analysis, called demand-driven (DD) analysis, assumes demands as pressure-independent known functions of time. DD analysis performs inadequately in abnormal conditions (Seyoum & Tanyimboh 2016).

A PDD analysis incorporates the relationship between demand and pressure. If some pressure is available above the minimum required level, it is assumed that a portion of demand will be supplied at the node. Thus, the PDD predicts better network performance under abnormal conditions, considering both nodal and pressure requirements. This study includes a pressure-demand relationship proposed by [Wagner \*et al.\* \(1988\)](#) for the PDD analysis, as shown in the equations below:

$$Q_{j,avl} = Q_{j,req} \quad \text{if } H_{j,avl} \geq H_{j,req} \quad (1a)$$

$$Q_{j,avl} = Q_{j,req} \left( \frac{H_{j,avl} - H_{j,min}}{H_{j,req} - H_{j,min}} \right)^{1/2} \quad \text{if } H_{j,min} < H_{j,avl} < H_{j,req} \quad (1b)$$

$$Q_{j,avl} = 0 \quad \text{if } H_{j,avl} < H_{j,min} \quad (1c)$$

The proposed optimization model incorporating PDD analysis has two objectives: minimization of cost and maximization of resilience.

### Minimization of cost

The first objective of this work is to minimize the network cost – the network cost increases in proportion to the scale of its components. Hence, the optimal size of the pipe diameters for the required pressure and demand must be chosen to achieve the best design. The network cost can be calculated as:

$$C = \sum_{i=0}^{np} C(D_i) * L_i \quad (2)$$

### Maximization of resilience

Various RIs have been developed in the past based on the energy flows within a network. Todini's RI ([Todini 2000](#)) indirectly measures the energy at nodes in terms of surplus head, which can be utilized during critical operating conditions ([Mangalekar & Gumaste 2021](#)). In the present work, MRI, introduced by [Jayaram & Srinivasan \(2008\)](#), is used to calculate resilience, which considers multiple sources in a network.

$$MRI = \frac{\sum_{i=1}^{nn} Q_{j,avl} (H_{j,avl} - H_{j,req})}{\sum_{i=0}^{nn} Q_{j,req} H_{j,req}} \quad (3)$$

### Normalized objective function

'Both objectives are evaluated by combining them with appropriate weightage. The combined objective function can be achieved by either assigning equal weights to each objective or varying weights. The importance anticipated by the water engineer or decision-maker can also be used to select appropriate weightage. Whenever a weightage-based technique is employed, it is critical to normalize each objective function to scaled-down various units of objective function values' ([Suribabu 2017](#)). The weighted function of normalized cost and resilience of the network is presented in the following formula as a single objective function:

$$\text{Min } (Z) = w \frac{C - C_{min}}{C_{max} - C_{min}} - (1 - w) \frac{MRI - MRI_{min}}{MRI_{max} - MRI_{min}} \quad (4)$$

The minimization of the above objective function is carried out satisfying the following conditions:

### Continuity of flow

For each node, continuity of flow must be met:

$$\sum Q_{in} - \sum Q_{out} = Q_{j,req} \quad (5)$$

$Q_{in}$  is the discharge going towards a specific node;  $Q_{out}$  is the discharge from the specific node.

### Energy conservation

The conservation of the energy equation for each loop should satisfy the network design:

$$\sum_{i=1}^{npl} H_f = 0 \quad (6)$$

$H_f$  is calculated using the Hazen-Williams formula, as shown in the equation below.

$$H_f = \frac{10.667 * L_i * Q_{i,avl}^{1.852}}{C_{hw}^{1.852} * D_i^{4.871}} \quad (7)$$

### Available pipe sizes

The diameter of the pipes should be selected from a set of commercially available sizes and are thus discrete:

$$D_i \in \{D_1, D_2, \dots, D_{ns}\}, \forall i \in ns \quad (8)$$

where  $D_1, D_2, \dots, D_{ns}$  are the commercially available pipe sizes.

### Minimum and maximum nodal pressure

At each node, the available pressure head should be greater than the minimum pressure head requirement and less than the maximum allowable pressure head.

$$H_{j,req} \leq H_{j,avl} \leq H_{j,max}, \forall j \in nn \quad (9)$$

### Minimum and maximum pipe-flow velocity

The flow velocity in each pipe should be greater than the minimum required flow velocity and less than or equal to the allowable maximum flow velocity.

$$V_{k,min} \leq V_{k,avl} \leq V_{k,max}, \forall k \in np \quad (10)$$

## MORAO ALGORITHM

The MORao algorithm is a multi-objective framework proposed to achieve resilient designs of WDNs based on the Rao algorithm. Rao algorithms were recently introduced by Rao (2020). ‘The main advantage of Rao algorithms is elimination of tuning algorithm-specific parameters to search optimal solutions. Its working procedure is clear to understand, easy to execute and straightforward’ (Rao & Pawar 2020). The working principle of the Rao algorithm is moving closer to the best solution and going away from the worst solution. The solution moves within the entire range of population with random interactions.

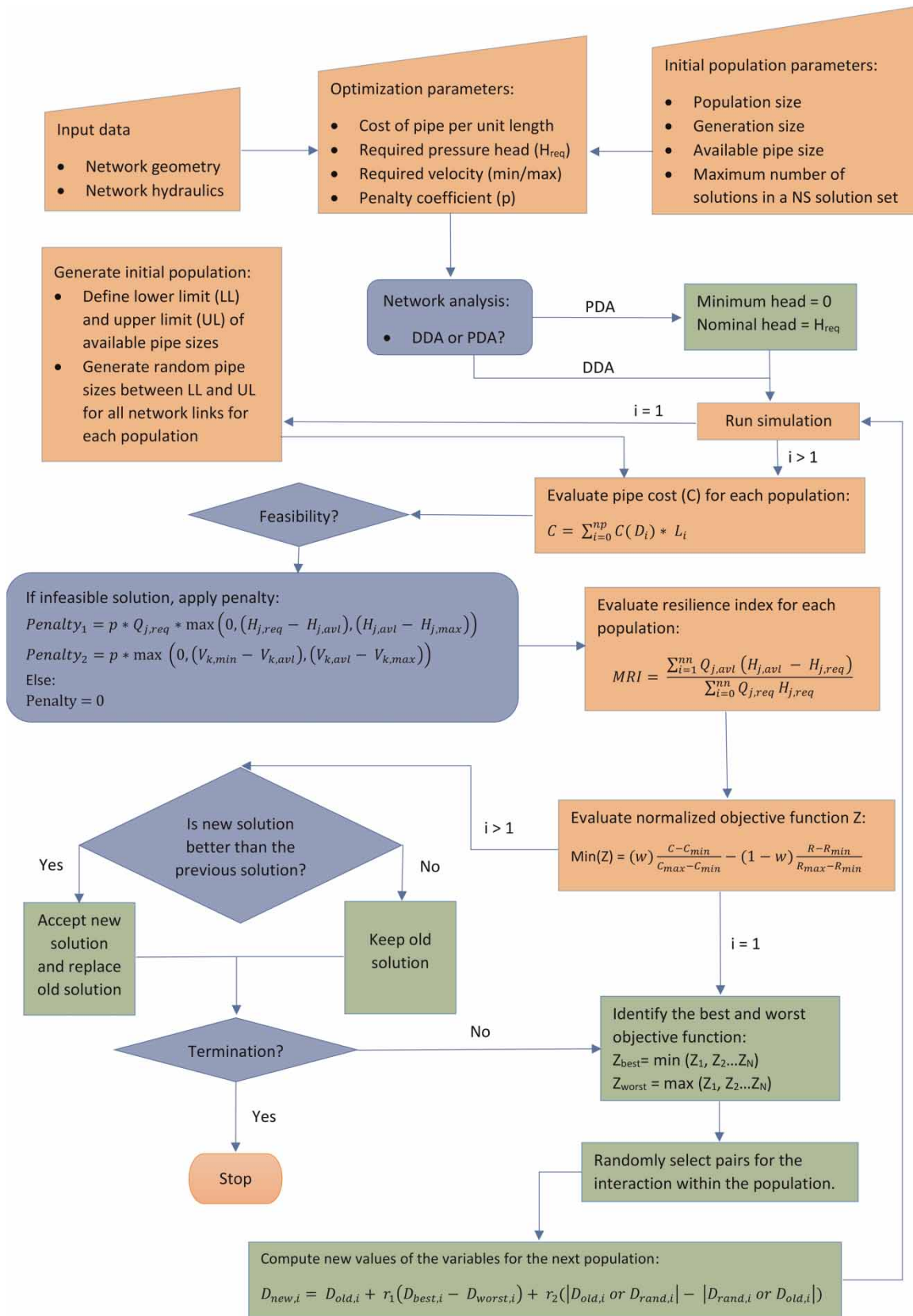
Figure 1 shows the basic procedure of the MORao algorithm using PDD analysis for the resilience-based optimal design of WDNs. First, the initial population of function variables (pipe diameters) is generated using random numbers with upper and lower bound values. The cost function is determined using these function variables using Equation (2). The feasibility of a specific solution is examined once the cost has been calculated. The feasibility criteria are fulfilling the required pressure head and demand at each node and the flow velocity in each pipe. A penalty is imposed on the cost function if an infeasible solution is identified. Equations (11) and (12) show the mathematical formulation of the penalty functions.

$$Penalty_1 = p * Q_{j,req} * \max(0, (H_{j,req} - H_{j,avl}), (H_{j,avl} - H_{j,max})) \quad (11)$$

$$Penalty_2 = p * \max(0, (V_{k,min} - V_{k,avl}), (V_{k,avl} - V_{k,max})) \quad (12)$$

After assessing feasibility, Equation (3) is used to calculate the resilience for each population. Subsequently, Equation (4) computes the normalized objective function. The best and worst candidates are selected from the range of solutions. The





**Figure 1** | Flowchart of MORao algorithm using PDD analysis for the resilience-based optimal design of a WDN.

variables corresponding to the best and worst solutions are used to generate a new population using one of the three equations of the Rao algorithms. 'In the Rao algorithms, the Rao-1 algorithm improves the result by considering the difference between the best and worst solutions; the Rao-2 and Rao-3 algorithms improve the result by considering not only the difference between the best and worst solutions but also the random interactions of the candidate solutions. The difference between Rao-2 and Rao-3 algorithms lie in considering the absolute values of the variables in the respective equations' (Rao & Pawar 2020). Rao-2 algorithm promotes more diversity because of the random interactions. Also, absolute values of variables should be considered to find the resilient solution as the value of pipe diameter is bound to be a real number. Therefore, the Rao-2 algorithm is applied in the present work, as shown in Equation (13). Finally, the previous and new objective functions are compared to find a better population for the next iteration.

$$D_{new,i} = D_{old,i} + r_1(D_{best,i} - D_{worst,i}) + r_2(|D_{old,i} \text{ or } D_{rand,i}| - |D_{rand,i} \text{ or } D_{old,i}|) \quad (13)$$

## CASE STUDY NETWORKS

The methodology is applied to three well-known benchmark networks. Figure 2 depicts the layout of all network cases. The basic network details of all cases are provided in the supplementary data.

### Two-loop network (TLN)

Alperovits & Shamir (1977) firstly used a two-loop network in their research study for the optimal design of a WDN using linear programming (LP). Later, various researchers adopted this network as a benchmark network (e.g., Todini (2000), Prasad & Park (2004) and, Sirsant & Reddy (2020)) in the field of optimization. It consists of one reservoir, six demand nodes, and eight links. The water is supplied using gravity (Wang *et al.* 2015).

### Goyang network (GYN)

Kim *et al.* (1994) introduced the Goyang network in South Korea, and it was solved using a projected Lagrangian algorithm. The GYN is also adopted as a benchmark problem in many pieces of research (Geem 2006; Wang *et al.* 2015). It is a pumped WDN and its hydraulic data comes from Geem (2006). This network consists of 22 nodes with one reservoir, 30 links, nine loops and a pump with a 71 m fixed reservoir head.

### Fossolo network (FOS)

The Fossolo network is named after a neighbourhood in Bologna, Italy called Fossolo. Like TLN and GYN, this network is a benchmark problem used in numerous WDN optimization works. The FOS consists of one reservoir with a fixed head of 121 m, 36 demand nodes and 58 links. The maximum pressure head of each node is specified in supplementary data. In addition, each pipe must have a flow velocity of less than or equal to 1 m/s (Bragalli *et al.* 2008).

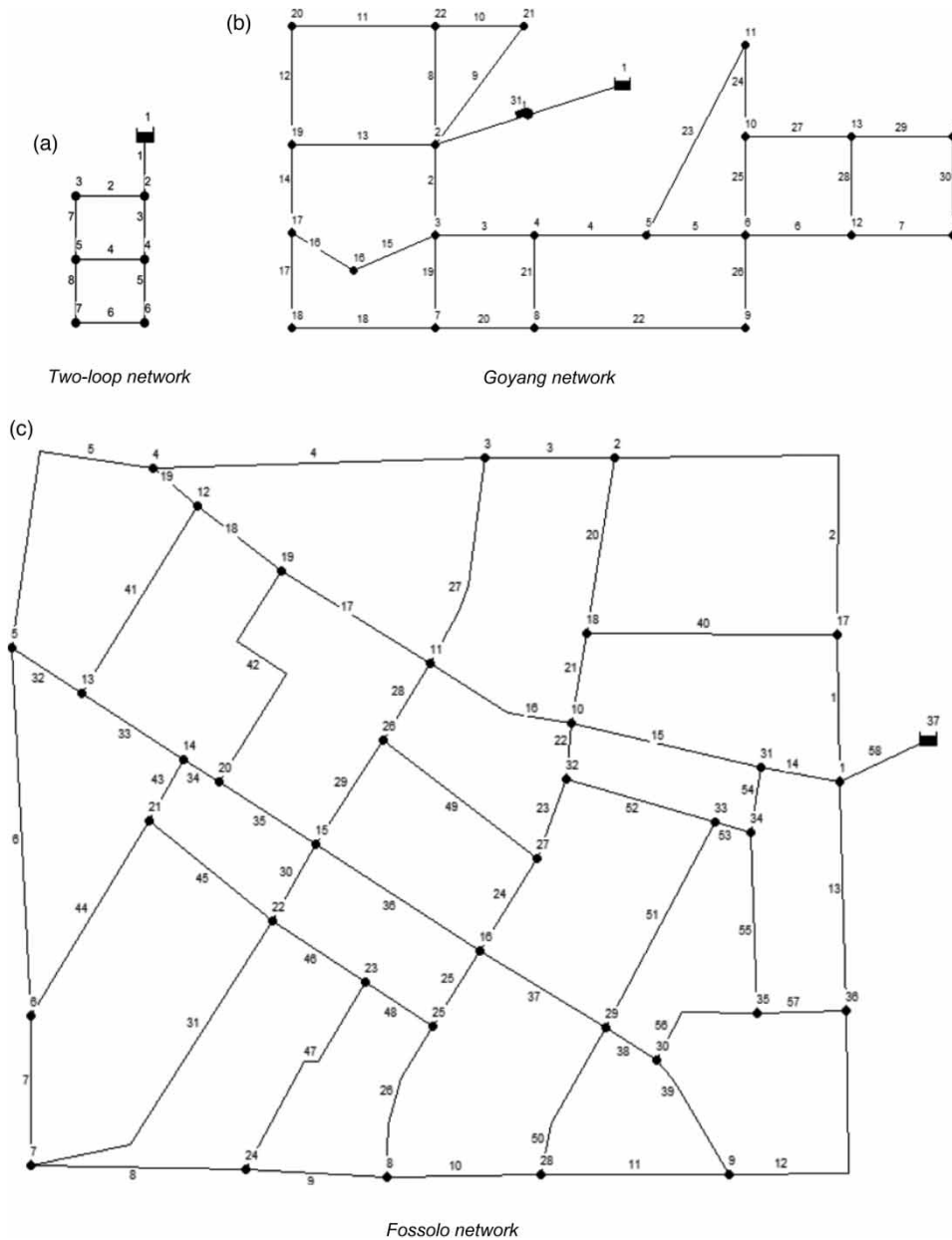
For the TLN, GYN and FOS networks, the problem complexity is considered as small, medium, and intermediate, respectively (Wang *et al.* 2015). The value of  $C_{hw}$  is 130 for TLN, 100 for GYN and 150 for FOS. The minimum pressure head requirement for TLN is 30 m, GYN is 15 m and FOS is 40 m. The available pipe diameters and corresponding unit costs for all case study networks are given in Table 2.

## RESULTS AND DISCUSSIONS

This paper demonstrates that the MORao algorithm can effectively and efficiently identify several high-quality alternatives for the resilient design of WDNs. Weights ranging from zero to one with an increment of 0.01% are employed to generate Pareto curves, as shown in Figure 3. These curves are plotted for all the case studies by keeping both population size and number of iterations as 100. In only 10,000 function evaluations, smooth curves are formed for small and medium-sized networks where smoothness of the curve depicts the evidence of non-dominated solutions.

A generational distance (GD) is measured for the approximate Pareto curve from the true PF. GD is a parameter used to compute the mean distance between a given set of solutions and the true PF (Tanyimboh & Seyoum 2020). GD determines the error between approximate PF and true PF. Since the true PF for the MRI against cost is not available in the literature, the Pareto fronts measured by Wang *et al.* (2015) against NRI are used. The simulated results of this work (i.e., head and discharge at each node) are used to calculate the NRI of each solution. Then, the PF of NRI against cost for the present





**Figure 2** | Layout of case study networks with node and link identifiers: (a) Two-loop network, (b) Goyang network, and (c) Fossolo network.

results is compared with PF measured by Wang *et al.* (2015) to calculate the GD. Table 3 shows network details (such as constraints and search space), numerical parameters of the MORao algorithm and calculated GD for all network cases. The GD was measured as 0.020 for TLN, 0.007 for GYN and 0.040 for FOS. It shows that the proposed approach provided an average accuracy of 97 percent to determine true PF. If the number of function evaluations is increased, the accuracy can further increase.

Subsequently, an investigation is conducted to experimentally determine the convergence of MORao algorithm to the true PF. The number of function evaluations (NFE) is increased to check if the simulated solution set moves closer to the true PF. Four different NFEs (100, 1,000, 10,000, 100,000) are used to generate four solution sets, as shown in Figure 4. For TLN, GYN and FOS, solutions are in line with PF in 1,000, 10,000 and 100,000 NFEs, respectively. With the increase in complexity of the network, like size of network, search space and number of constraints, required computational efforts are increased.

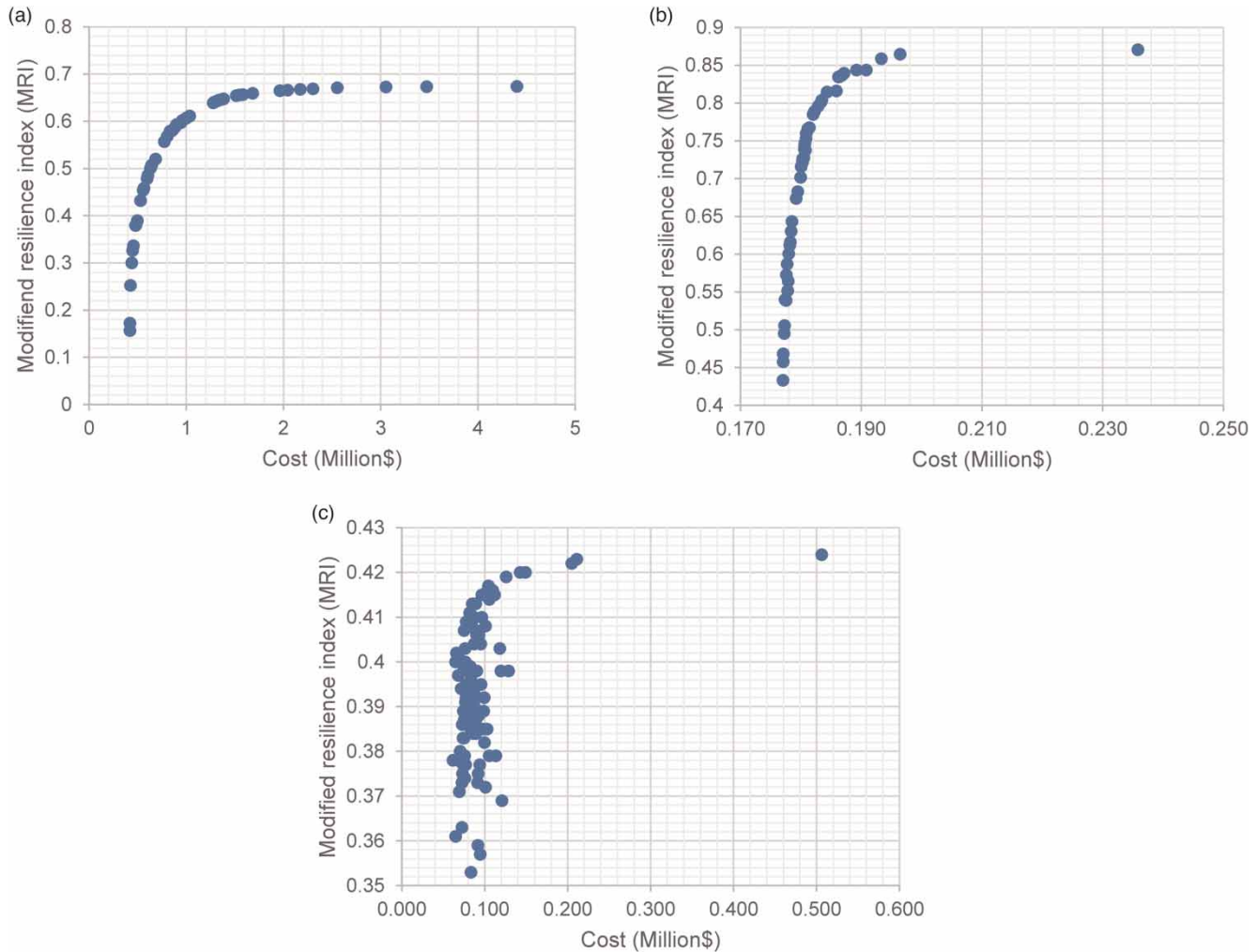
**Table 2** | Available pipe sizes for all network studies

Two-loop network <b>Alperovits &amp; Shamir (1977)</b>		Goyang network <b>Geem (2006)</b>		Fossolo network <b>Bragalli et al. (2008)</b>	
Diameter (mm)	Cost (\$/m)	Diameter (mm)	Cost (\$/m)	Diameter (mm)	Cost (\$/m)
25.4	2	80	37.890	16	0.38
50.8	5	100	38.933	20.4	0.56
76.2	8	125	40.563	26	0.88
101.6	11	150	42.554	32.6	1.35
152.4	16	200	47.624	40.8	2.02
203.2	23	250	54.125	51.4	3.21
254	32	300	62.109	61.4	4.44
304.8	50	350	71.524	73.6	6.45
355.6	60			90	9.59
406.4	90			102.2	11.98
457.2	130			114.6	14.93
508	170			130.8	19.61
558.8	300			147.2	24.78
609.6	550			163.6	30.55
				184	38.71
				204.6	47.63
				229.2	59.70
				257.8	75.61
				290.6	99.58
				327.4	126.49
				368.2	160.29
				409.2	197.71

Particularly in the FOS, a higher level of constraints is imposed, consisting of maximum allowable pressure limit and maximum allowable flow velocity. Consequently, there is a possibility of achieving better solutions with larger NFEs. Therefore, this investigation also explains the absence of a non-dominated solution set for FOS in 10,000 NFEs, as shown in Figure 3. Hence, a smooth curve can be obtained if NFE is increased to 100,000.

If possible, one should select an appropriate design alternative from the calculated Pareto optimal set. Investors and consumers are two groups of stakeholders that try to decrease and increase pipe diameters to minimize cost and maximize reliability, respectively. These conflicts can make the selection of an appropriate alternative difficult. The Nash (1950) bargaining model was thus used to select a compromise solution from the Pareto optimal sets. For this, it is necessary to know the utility function of various stakeholders to make a good decision. In this work, three benchmark networks are considered without real stakeholders to make decision. As proposed by Beygi et al. (2014), implicit utilities are used which resulted 0.3, 0.5 and 0.7 weight (w) alternatives.

Table 4 shows the cost and MRI values of the preferred alternatives. When more significant weightage is assigned to the normalized objective function of MRI, the resultant solution has a higher MRI value and a higher cost, and vice versa. In Figure 5, solution 1 corresponds to the minimum cost design accomplishing the first objective function; solution 2 corresponds to maximum resilience design accomplishing the second objective function; solutions 3, 4 and 5 are the compromise solutions fulfilling both objectives and providing appropriate alternatives to multi-objective design. If solution 1 is selected, there is a high probability that consumer will not agree to the decision, while an investor will not agree for to solution 2. It is significant that solutions 3, 4 and 5 are the most probable solutions selected from the 100 Pareto optimal



**Figure 3** | Approximate Pareto curve for resilience-based optimal design using MORao algorithm: (a) Two-loop network, (b) Goyang network and (c) Fossolo network.

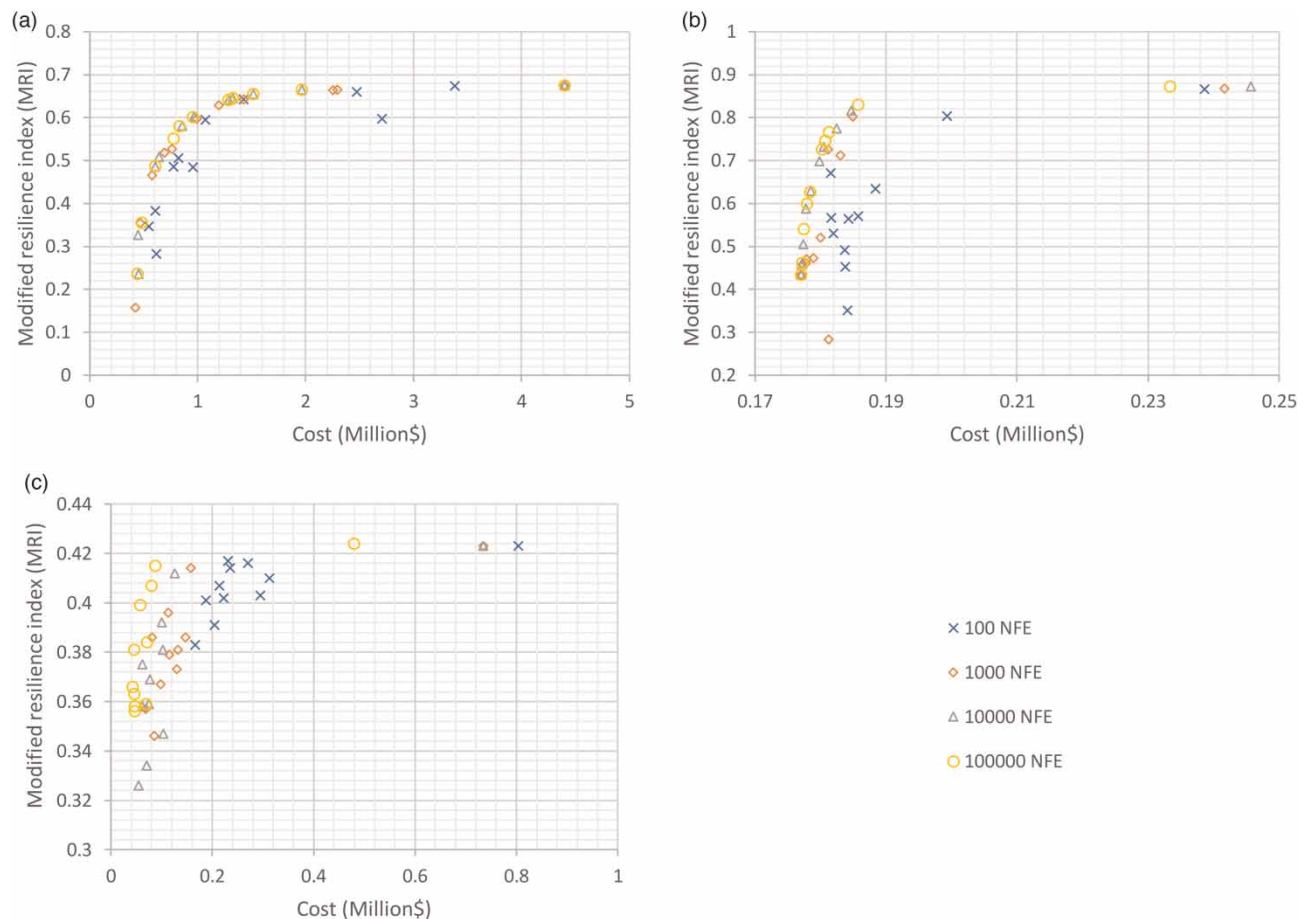
solutions – fulfilling the utilities of both stakeholders and giving decision-makers the flexibility to choose. Out of these three solutions, for all network cases, solution 5 looks more promising considering the combined effect of reduction in cost concerning maximum resilient design and increment in resilience concerning minimum cost design.

## CONCLUSIONS

Rao algorithms have not been used in the past for multi-objective optimization. Nevertheless, its non-parametric characteristic made it more striking to be used in highly complex problems of WDN having non-linear constraints and multi-objectiveness. Three WDNs of small (TLN), medium (GYN) and intermediate (FOS) size and complexity were used as benchmark networks to demonstrate the efficiency of the MORao algorithm in finding resilience-based optimum design. Present work provides a simple technique of combining more than one objective and generating the PF with a designated number of solutions. A set of legitimate solutions is acquired by changing the weights for both cost and MRI. A high-quality tradeoff between cost and resilience is attained with 97% proximity to the true PF. The Nash bargaining model was used to choose the optimal alternative designs. Because of the difference in characteristics of the considered case studies, variation in results is observed. However, all results were acceptable with variation in profitability of the investors and consumers. The choice of the solution will highly depend on the weights given to the cost and resilience. For TLN, GYN and FOS, if more weight is given to the network cost without ignoring the consumer's utility, solution 4 can be considered the best alternative design,

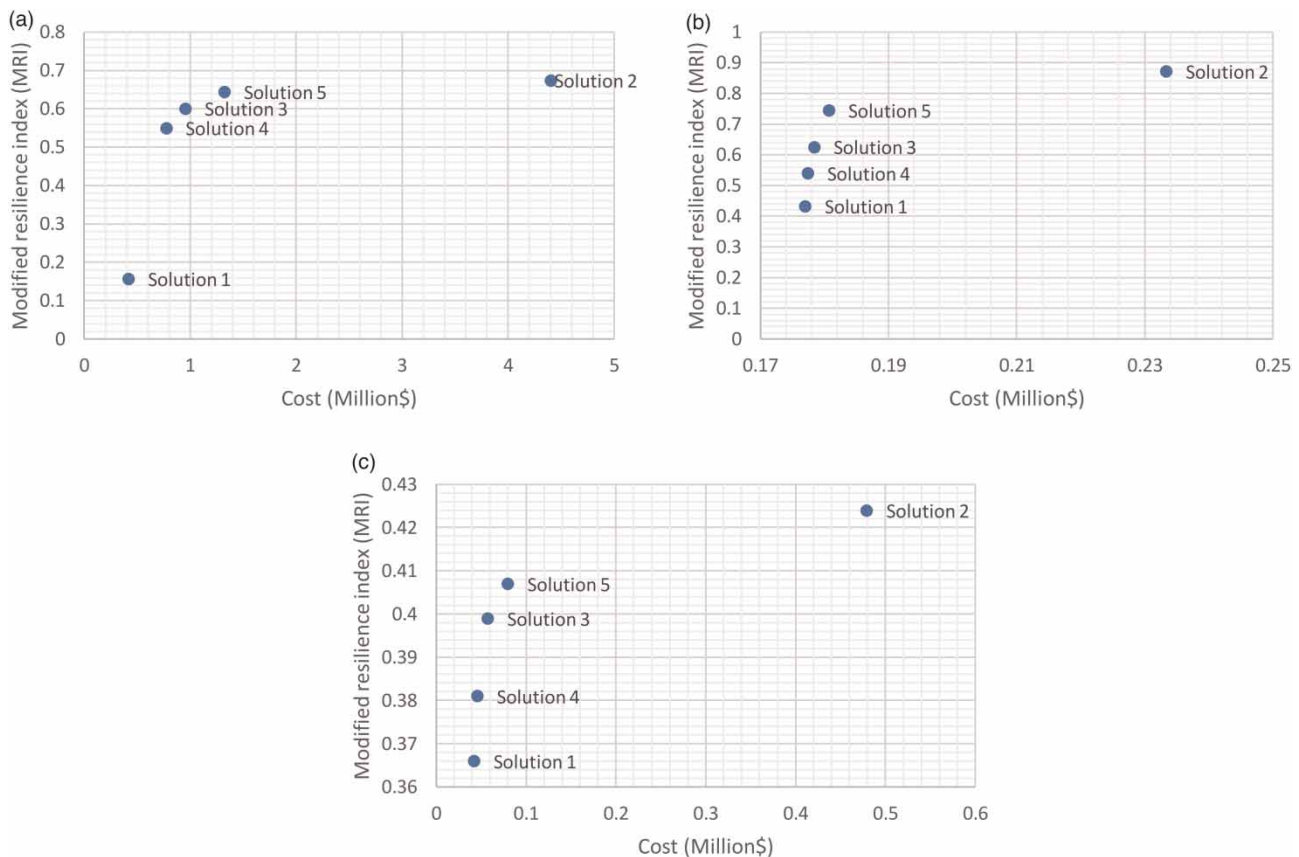
**Table 3** | Network details and numerical parameters

Benchmark network	Two-loop network	Goyang network	Fossolo network
Reference	Alperovits & Shamir (1977)	Geem (2006)	Bragalli <i>et al.</i> (2008)
No. of decision variables	8	30	58
No. of available diameters	14	8	22
Size of total search space	$1.48 \times 10^9$	$1.24 \times 10^{27}$	$7.25 \times 10^{77}$
Minimum pressure constraint	30 m	20 m	40 m
Maximum pressure constraint	–	–	Variable <sup>a</sup>
Velocity constraint	–	–	$V < 1$ m/s
Number of Pumps	0	1	0
Population size	100	100	100
Number of generations	100	100	100
Function evaluations	10,000	10,000	10,000
Penalty coefficient ( $\lambda$ )	6	10	8
Weightage increment size	0.01	0.01	0.01
Generational distance	0.020	0.007	0.042

<sup>a</sup>Details are given in supplementary data.**Figure 4** | Convergence of MORao algorithm with the number of function evaluations: (a) Two-loop network; (b) Goyang network; (c) Fossolo network.

**Table 4** | Optimal solutions selected from the Pareto optimal set

Benchmark network	Bargaining model	Optimal solutions				
		Solution number	Cost (\$)	MRI	Reduction in cost (%)	Increment in MRI (%)
TLN	Least cost design	1	419,000	0.157	90.48	0
	Maximum Resilience design	2	4,400,000	0.674	0	329.30
	Nash $w = 0.5$	3	954,000	0.601	78.32	282.80
	$w = 0.7$	4	774,000	0.550	82.41	250.32
	$w = 0.3$	5	1,324,000	0.644	69.91	310.19
GYN	Least cost design	1	177,010	0.433	24.14	0
	Maximum Resilience design	2	233,323	0.872	0	101.39
	Nash $w = 0.5$	3	178,389	0.626	23.54	44.57
	$w = 0.7$	4	177,398	0.54	23.97	24.71
	$w = 0.3$	5	180,685	0.745	22.56	72.06
FOS	Least cost design	1	42,012	0.366	91.85	0
	Maximum Resilience design	2	479,212	0.424	0	15.85
	Nash $w = 0.5$	3	57,325	0.399	88.04	9.02
	$w = 0.7$	4	45,689	0.381	90.47	4.10
	$w = 0.3$	5	79,760	0.407	83.36	11.20

**Figure 5** | Solutions recognized using Nash bargaining model from the Pareto optimal sets: (a) Two-loop; (b) Goyang; (c) Fossolo network.



with \$774,000, \$177,398 and \$45,689 optimal costs. However, solution 5 gives more resilient design with \$1,324,000, \$180,685 and \$79,760 optimal costs. Being easy to use and understand, designers and researchers can apply this methodology to find the resilient design of a WDN of any size or characteristic by only modifying the penalty coefficient as per the size of the network. In the future, other reliability measures can also be applied in place of MRI or combined with MRI, like redundancy or entropy in this technique.

## DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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