


## Prediction of hydraulic conductivity of porous media using a statistical grain-size model

Abhishish Chandel , Shivali Sharma  and Vijay Shankar 

National Institute of Technology, Hamirpur, India

\*Corresponding author. E-mail: abhishishchandel@gmail.com

 AC, 0000-0002-3082-2721; SS, 0000-0002-2690-4222; VS, 0000-0002-9509-6804

### ABSTRACT

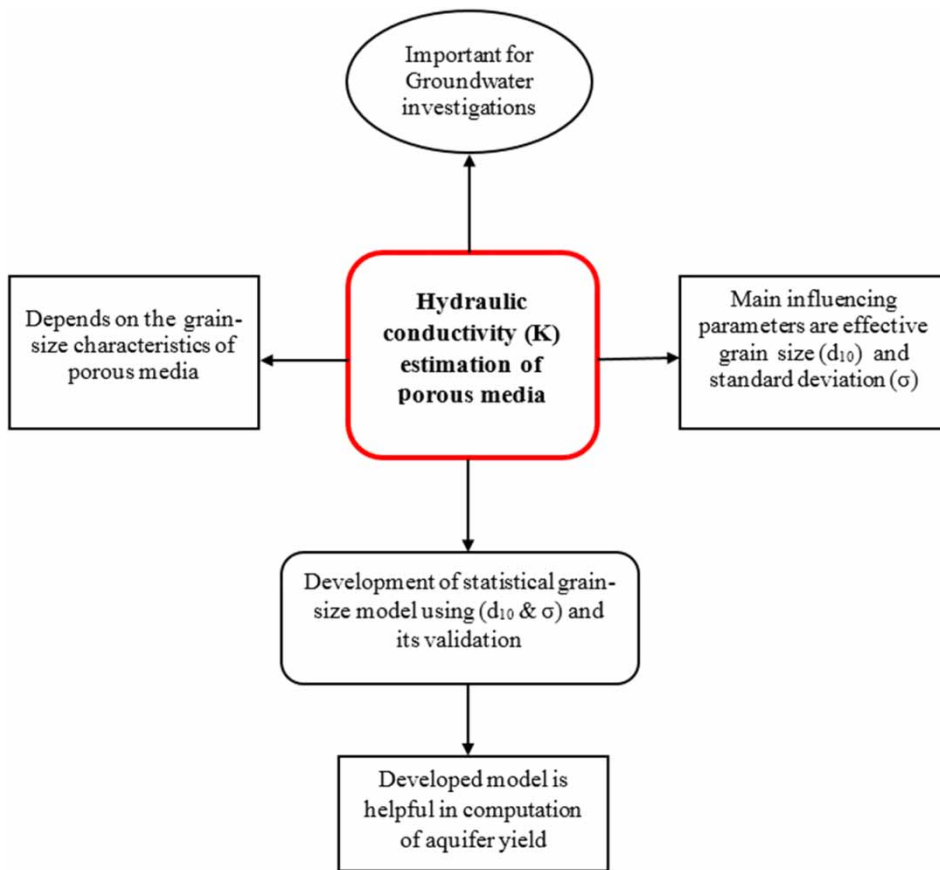
Hydraulic conductivity (K) estimation of porous media is of great significance in contaminant movement and groundwater investigations. The present study examines the influence of effective grain size ( $d_{10}$ ) and standard deviation ( $\sigma$ ) on the K value of borehole soil samples using 5.08, 10.16, and 15.24 cm diameter permeameters. A statistical grain size model was developed and the feasibility of seven empirical equations was evaluated with the measured K values. The K of soil samples increases with the increase in the  $d_{10}$  grain size and decreases with the increase in the  $\sigma$  value. Evaluation of K using empirical equations establishes that the Hazen equation shows relatively good agreement with the measured K values. The study substantiates the efficacy of the developed model as the  $K_{\text{model}}$  and  $K_{\text{measured}}$  based  $R^2$  (determination coefficient), MAE (mean absolute error), and RMSE (root mean square error) values are (0.982, 0.007, and 0.008), (0.972, 0.005, and 0.007), (0.953, 0.004, and 0.005) for 5.08, 10.16, and 15.24 cm diameter permeameters respectively. The developed model was validated by assessing its efficiency in the prediction of K values for independent soil samples. The developed model-based K accedes to the precise computation of the aquifer yield and groundwater recharge.

**Key words:** effective grain size, hydraulic conductivity, porosity, porous media

### HIGHLIGHTS

- The study proposes a statistical grain size model for the computation of hydraulic conductivity of porous media by investigating the influence of the  $\sigma/d_{10}$  parameter on hydraulic conductivity.
- The developed hydraulic conductivity model provides an efficient tool to compute the aquifer yield, groundwater recharge, and filter design with precise accuracy.

## GRAPHICAL ABSTRACT



## INTRODUCTION

Understanding the concept of hydraulic conductivity ( $K$ ) is a fundamental objective for hydrologists in groundwater and geotechnical investigations, and management practices (Pliakas & Petalas 2011; Pucko & Verbovsek 2015). The concept of  $K$  was first postulated by Henry Darcy and defined as the ease with which the fluid flow can take place through the interconnected voids (Wang *et al.* 2017). Darcy's equation correlates the fluid velocity with the hydraulic gradient. The proportionality constant in that equation is termed as the hydraulic conductivity of porous media (Kasenow 2002; Ghanbarian *et al.* 2016). The computation of dimensionless fluid quantities namely friction factor and Reynolds number of porous media is important to govern the flow regime (Li *et al.* 2019). The  $K$  of the porous media is dependent invariably on the fluid properties i.e., viscosity and specific weight, and the characteristics of the porous media which include the particle size, structural configuration, grain size distribution, and compaction of soil particles (Zieba 2017; Chandel *et al.* 2021). The investigations regarding the prediction of  $K$  of porous media are important to determine the transmitting capacity of the soil particles or the bed, as these factors are dependent on the aquifer recharge or extraction (Cronican & Gribb 2004; Zhu *et al.* 2021).

For computing the  $K$  of porous media, there exist several methods namely field and laboratory methods, and empirical equations (Alabi 2011). The field methods involve cost factors and long testing time which makes them a less reliable approach. The laboratory methods also sometimes prove to be uneconomical due to time restrictions and cost (Boadu 2000). In addition to this, it is challenging to collect the porous media samples representing the actual field conditions for experimental investigations and for field methods, the unavailability of the precise knowledge of the boundary and the aquifer geometry can be a restrictive factor (Riha *et al.* 2018). However, the prediction of  $K$  using empirical equations is a reliable approach for the rough computation of  $K$  via grain size parameters because the data related to the textural characteristics of the soil particle are effortlessly and quickly obtained (Rosas *et al.* 2014; Chandel & Shankar 2021). Moreover, numerous groundwater researchers have focused to establish a relationship between  $K$  and grain size parameters, as such proposed

relationships are not dependent on the aquifer boundaries and relatively less expansive as compared to other techniques (Lu *et al.* 2012; Cabalar & Akbulut 2016).

Various researchers in the past such as Vienken & Dietrich (2011), and Banerjee *et al.* (2019) examined different modelling techniques for computing the K value of porous media. These studies presented elucidations having some limitations i.e., simulating the infinite extent of subsurface flow (Zahiri & Najafzadeh 2018). The one crucial limitation is to control other parameters that affect the K value of porous media while investigating the influence of one particular parameter i.e., effective and mean size ( $d_{10}$  and  $d_{50}$ ), standard deviation about the mean, and dispersion measure ( $d_{50}-d_{10}$ ) (Boadu 2000; Song *et al.* 2009). Krumbein & Monk (1943) stated that the K can be expressed as the product of the exponential and power function of the standard deviation and mean size respectively. Alyamani & Sen (1993) investigated the influence of the grain size parameter on the K of porous media. From this study, it was postulated that the finer grain zone within the grain size curve is vital in the prediction of hydraulic conductivity. Barr (2001) proposed an equation based on Darcy's concept to predict the K of porous sediments. The proposed equation is composed of the parameters which can be computed directly, rather than assuming i.e., shape factor, fluid viscosity, and porosity. Odong (2007) and Ishaku *et al.* (2011) examined the K of porous media using various grain size-based empirical equations and concluded that the empirical equations should be applied within particular domains of applicability. Pliakas & Petalas (2011) examined the correlation between the statistical grain size parameters with the computed K values. Salarashayeri & Siosemarde (2012) developed an equation based on multiple regression analysis using grain diameters i.e.,  $d_{10}$ ,  $d_{50}$ , and  $d_{60}$ , and postulated that grain diameter ( $d_{10}$ ) proved to be an effective parameter in computing the K of porous media. Pucko & Verbovsek (2015) compared different techniques such as empirical grain size and pumping methods to estimate the K of porous media. The study concluded that the grain size methods result in the precise prediction of K compared to the field techniques. Naeef *et al.* (2017) proposed an equation to estimate the K of porous media using M5 model regression analysis. The M5 model is the binary decision tree, which includes the linear regression equations. Further, the K values from the proposed equation were evaluated with the values calculated from empirical equations and concluded that the developed equation performs better in computing the K of porous media. Wang *et al.* (2017) proposed a new equation based on the gradation characteristics namely effective grain size ( $d_{10}$ ), porosity, and uniformity coefficient to predict the K of porous media. Arshad *et al.* (2020) concluded that the effective grain size ( $d_{10}$ ) substantially gives better results in the K prediction using empirical models as compared to the mean grain size ( $d_{50}$ ). Existing literature establishes that various researchers correlated the K with various influencing parameters instead of the standard deviation and effective grain size of porous media. The effective grain size represents the entire characteristics of the grain size curve, whereas the standard deviation measures the precise scatter of grain size values from the mean particle size. Moreover, in the previous studies, the developed equations to predict the K are valid for a small range of grain size parameters.

Based on the above discussion, the present study emphasizes the effect of grain size distribution, which has a substantial influence on the K of the porous media. The two parameters namely effective grain size ( $d_{10}$ ) and standard deviation ( $\sigma$ ) have been integrated in the investigation, which are easily computed for any porous media and has, significant impact on the behavior of grain size distribution. The primary objectives of the study are:

- (1) To study the variation between the Friction factor (Fr) and Reynolds number (Re) for different sizes of porous media to govern the flow regime.
- (2) To study the influence of standard deviation ( $\sigma$ ) and effective grain size ( $d_{10}$ ) on experimentally measured K of porous media.
- (3) To develop a model for computing the K of porous media, involving grain size parameters ( $\sigma$  and  $d_{10}$ ), and assess the efficacy of the developed model vis-a-vis the pre-existing empirical equations based on the comparative evaluation of the computed and measured K values.
- (4) To validate the developed model for computing the K for a wide range of porous media.

## MATERIALS AND METHODS

### Materials

In the present work, 27 representative borehole soil samples were collected from the Kangra district of Himachal Pradesh in India. The soil samples were acquired from an ongoing drilling operation, which was established to locate the aquifer geological profile. In order to obtain the undisturbed soil samples, thin-walled sampler tubes having a diameter and length of

8.5 and 115 cm respectively were used, so that the collected samples are subjected to minimum disturbance concerning the actual field conditions. The study area has a ground elevation of 425–3,500 m above the mean sea level and lies in the north-west region of the Himalayas. The latitude and longitude of the study area are 31°3'N and 75°25'E respectively. The soil characteristics of the study area comprise sand, gravel, and silts. During the drilling operation, three soil samples from nine boreholes were collected, which were drilled at an interval of 5–6 m apart. The soil samples were collected at an interval of 3 m from the core material and then subjected to dry sieve analysis in the laboratory for further experimental investigations.

## Methodology

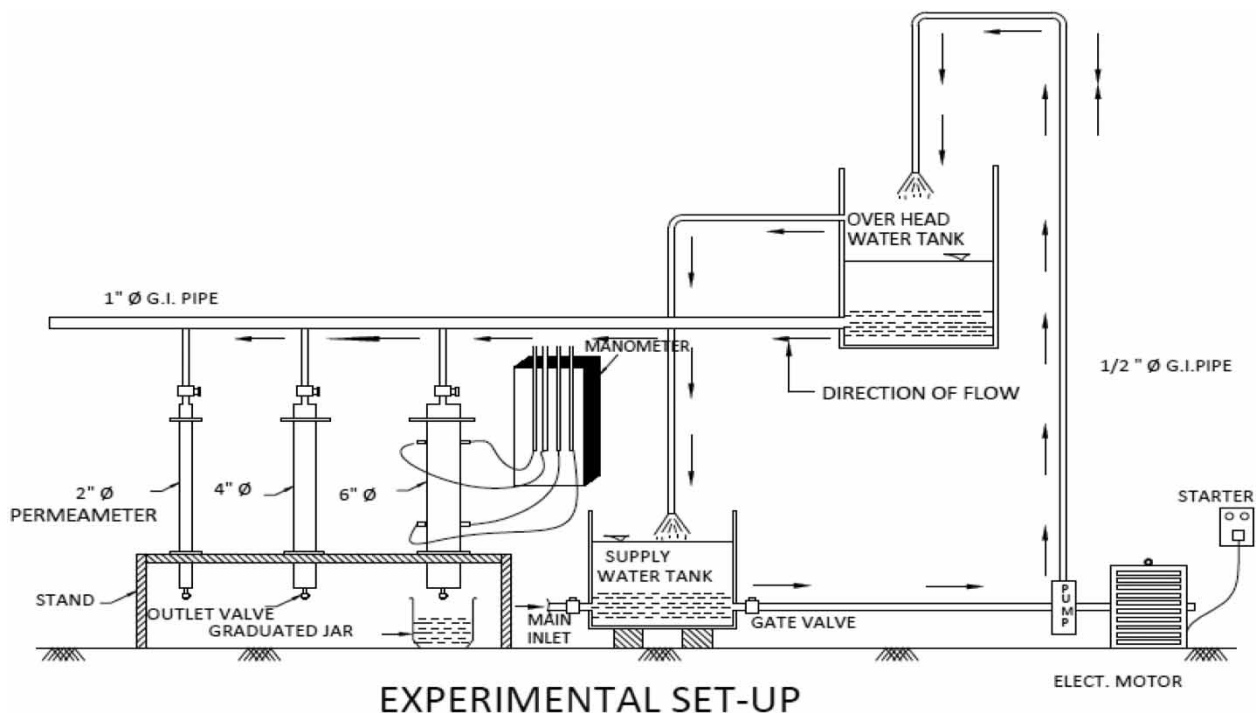
Initially, the collected soil samples were subjected to dry sieve analysis using a mechanical sieve shaker device to determine the effective and mean size of each sample. The specific gravity was computed using the standard pycnometer method, which is imperative for the porosity determination. The effective size  $d_{10}$  is specified by standard guidelines using sieve analysis and the value of standard deviation is computed from the relation (Todd & Mays 2005) as:

$$\sigma = \left( \sum (d_{50} - d_i)^2 * \Delta Z_i \right)^{0.5} \quad (1)$$

where,  $\sigma$  = standard deviation,  $d_{50}$  = grain size corresponding to 50% finer by weight,  $d_i$  = particle diameter retained on a specific sieve, and  $\Delta Z_i$  = fraction of total particles.

The resistance offered to the fluid flow through porous media was assessed by conducting the hydraulic test. The K of soil samples was estimated using a Constant Head Permeameter (CHP) having internal diameter i.e., 5.08, 10.16, and 15.24 cm as shown in Figure 1.

For estimating the K value, Figure 1 represents the line diagram of the experimental setup, which is established in the laboratory. The setup comprises permeameters which are manufactured from Galvanized iron pipe having a total and test length of 106 cm and 46.5 cm respectively. An overhead tank is located at a height of 2.65 m above the permeameter outlet. The overhead tank supplies the water to the permeameter and receives water supply from the re-circulating tank continuously to maintain a constant head level. Pressure tapping points are provided along the periphery of the permeameter at a center to center distance of 46.5 cm to measure the pressure difference readings. The arrangement of pressure tapping points



**Figure 1** | Experimental setup of K measuring apparatus.

helps in recording the manometer readings. The permeameter consists of an inlet and outlet pipe of 19 mm diameter each to regulate the water flow rate through the permeameter. The discharge measurement is done by collecting the water in a measuring cylinder for an appropriate time interval i.e., 30 sec via a digital stopwatch. For one sample 5–6 discharge readings have been recorded and the average of the discharge value is used to compute the hydraulic conductivity. The K value of soil samples was measured via the standard procedure as described by [Rosas \*et al.\* \(2014\)](#) and [ASTM \(2006\)](#). The water temperature was measured using a digital thermometer at the start and end of each analysis. The hydraulic conductivity of soil samples has been determined using Darcy's equation ([Qiu & Wang 2015](#)):

$$K = \frac{Q * L}{A * h} \quad (2)$$

where, Q = flow volume (m<sup>3</sup>/s), L = test length (m), h = head difference between the pressure taps (m), and A = sample cross-sectional area (m<sup>2</sup>).

In this study, the influence of  $d_{10}$  and  $\sigma$  on the K of 15 borehole soil samples was investigated individually. Further, the behaviour of hydraulic conductivity, obtained from three diameter permeameters have been analyzed with the different values of  $\sigma/d_{10}$ , which provides the basis for developing a statistical grain size model using the principle of least squares approach. For the model development, a data set of 45 hydraulic conductivity values have been used in the study.

### Empirical equations for K estimation

The prediction of K using empirical equations depends primarily on the viscosity, uniformity coefficient, grain size, porosity, and sorting coefficient. Numerous researchers have performed experimental investigations to develop the empirical equations by analyzing the interrelationship between these parameters. [Vukovic & Soro \(1992\)](#) and [Song \*et al.\* \(2009\)](#) have given a generalized hydraulic conductivity equation for various empirical equations:

$$K = \frac{g}{\nu} * \alpha * f(n) * d_e^2 \quad (3)$$

where, K = hydraulic conductivity (m/s),  $\alpha$  = dimensionless coefficient, depends on the various porous media parameters (grain size and shape, structure, and heterogeneity)  $\nu$  = kinematic viscosity (m<sup>2</sup>/s); g = acceleration due to gravity (m/s<sup>2</sup>),  $f(n)$  = porosity function, represents the degree of porous media compactness, and  $d_e$  = effective diameter (m).

Various researchers have proposed different empirical equations by analyzing the relationships of K with various influencing parameters i.e.,  $d_e$ ,  $f(n)$ , and  $\nu$ . Based on Equation (3), the empirical equations have been reoriented and written in the standard form. [Table 1](#) shows the empirical equations used in this study for the hydraulic conductivity evaluation with their applicability limits. Also, the two recently proposed empirical equations namely [Chapuis \*et al.\* \(2005\)](#) and [Naej \*et al.\* \(2017\)](#)

**Table 1** | Empirical equations for K prediction

Researcher	Equation	Applicability limits
<a href="#">Hazen (1892)</a>	$K = \frac{g}{\nu} * 6 \times 10^{-4} * [1 + 10(n - 0.26)] * d_{10}^2$	0.1 mm < $d_{10}$ < 3 mm, $U < 5$
<a href="#">Slichter (1899)</a>	$K = \frac{g}{\nu} * 1 \times 10^{-2} * n^{3.287} * d_{10}^2$	0.01 mm < $d_{10}$ < 5 mm
<a href="#">Terzaghi (1925)</a>	$K = \frac{g}{\nu} * 8.4 \times 10^{-3} * \left( \frac{n - 0.13}{\sqrt[3]{1 - n}} \right)^2 * d_{10}^2$	Large grain sand
Kozeny–Carman ( <a href="#">Kozeny 1927</a> ; <a href="#">Carman 1937, 1956</a> )	$K = \frac{g}{\nu} * 8.3 \times 10^{-3} * \left[ \frac{n^3}{(1 - n)^2} \right] * d_{10}^2$	Suitable for gravel and sand, $d_{10} < 3.0$ mm
<a href="#">Harleman <i>et al.</i> (1963)</a>	$K = \frac{g}{\nu} * 6.54 \times 10^{-4} * d_{10}^2$	Coarse and well grained media
<a href="#">Chapuis <i>et al.</i> (2005)</a>	$K = 1.412 * \frac{n^{2.35}}{(1 - n)^{1.565}} * d_{10}^{1.565}$	0.03 mm ≤ $d_{10}$ ≤ 3 mm
<a href="#">Naej <i>et al.</i> (2017)</a>	$K = \frac{g}{\nu} * 1.84 \times 10^{-4} * d_{10}^{0.83} * U^{-0.55}$	Valid for sand and gravel $d_{10} < 3.0$ mm

Where,  $n$  = porosity,  $U$  = uniformity coefficient i.e.,  $d_{60}/d_{10}$ .

have been incorporated for the hydraulic conductivity assessment. The  $g$  and  $\nu$  values in the empirical equations considered in this study were taken as  $981 \text{ cm/s}^2$  and  $0.885 \text{ mm}^2/\text{s}$  respectively.

### Statistical performance indicators

For quantitative assessment between the measured and computed  $K$  values, various statistical indicators i.e., BIAS, scatter index ( $S_i$ ), determination coefficient ( $R^2$ ), agreement index ( $I_a$ ), mean absolute error (MAE), and root mean square error (RMSE) have been used in this study (Naeef *et al.* 2017). The statistical indicators are defined as:

$$BIAS = \sum_{i=1}^Z \frac{1}{Z} (X_i - K_i) \quad (4)$$

$$S_i = \frac{\sqrt{\frac{1}{Z} \sum_{i=1}^Z (X_i - K_i)^2}}{\bar{K}_i} \quad (5)$$

$$R^2 = \left[ \frac{\sum_{i=1}^Z (K_i - \bar{K})(X_i - \bar{X})}{\sqrt{\sum_{i=1}^Z (K_i - \bar{K})^2 \sum_{i=1}^Z (X_i - \bar{X})^2}} \right]^2 \quad (6)$$

$$I_a = 1 - \frac{\sum_{i=1}^Z (X_i - K_i)^2}{\sum_{i=1}^Z |X_i - \bar{K}| + |K_i - \bar{K}|} \quad (7)$$

$$MAE = \frac{1}{Z} \sum_{i=1}^Z |K_i - X_i| \quad (8)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^Z (K_i - X_i)^2}{Z}} \quad (9)$$

where,  $Z$  is the number of datasets,  $K_i$  and  $X_i$  denote the measured and computed  $K$  values respectively.  $\bar{K}$  and  $\bar{X}$  represent the average values of measured and computed parameters, respectively.

## RESULTS AND DISCUSSION

Experimental investigations include the grain size analysis and CHP test, which have been performed on the collected borehole soil samples. A total number of 27 soil samples have been used i.e., 15 soil samples to study the influence of  $\sigma$  and  $d_{10}$  on  $K$  and for model development whereas, the remaining 12 soil samples were considered as independent samples for the validation of the developed model.

### Grain-size analysis

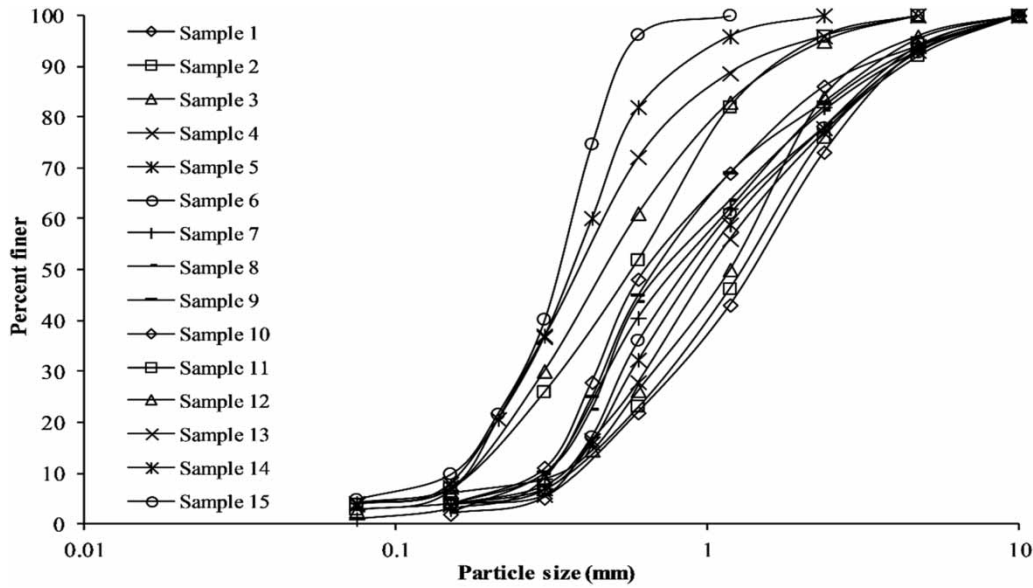
Initially, the grain size analysis was conducted on the collected borehole soil samples and then the grain size curve for the 15 soil samples was plotted between the percent finer and particle size as shown in Figure 2. From the grain size curve the grain sizes ( $d_{10}$ ,  $d_{30}$ ,  $d_{50}$ , and  $d_{60}$ ), uniformity coefficient, and standard deviation values were determined. For the remaining soil samples, the grain size parameters ( $\sigma$  and  $d_{10}$ ) have been determined and mentioned in the validation section.

The basic properties of the 15 soil samples is presented in Table 2. The values of  $d_{10}$  and  $\sigma$  vary between 0.173–0.386 mm and 1.470–7.100 respectively.

### Variation between friction factor and Reynolds number

The dimensionless flow parameters i.e.,  $F_r$  and  $R_e$  were computed for soil samples using permeameter with different diameters i.e., 5.08, 10.16, and 15.24 cm. A logarithmic plot has been drawn between the dimensionless flow parameters as shown in





**Figure 2** | Gradation curve of soil samples 1–15.

**Table 2** | Basic properties of the soil samples

sample no.	Gravel (%)	Sand (%)	Silt (%)	d <sub>10</sub> (mm)	d <sub>30</sub> (mm)	d <sub>50</sub> (mm)	d <sub>60</sub> (mm)	n <sup>a</sup>	U <sup>a</sup>	σ <sup>a</sup>
1	25.78	68.76	5.46	0.386	0.821	1.450	1.850	0.359	4.793	1.470
2	27.41	70.46	2.13	0.373	0.774	1.330	1.720	0.363	4.611	1.560
3	31.24	66.78	1.98	0.358	0.690	1.180	1.530	0.370	4.274	1.640
4	21.91	74.82	3.27	0.347	0.631	1.050	1.400	0.375	4.035	1.790
5	18.73	76.84	4.43	0.342	0.575	0.980	1.240	0.385	3.626	1.980
6	16.28	81.46	2.26	0.330	0.544	0.920	1.160	0.387	3.515	2.190
7	12.64	83.71	3.65	0.325	0.499	0.850	1.120	0.389	3.446	2.460
8	14.82	81.73	3.45	0.301	0.486	0.785	1.070	0.386	3.555	2.780
9	19.43	76.17	4.40	0.297	0.468	0.720	0.962	0.394	3.239	2.980
10	11.84	85.29	2.87	0.278	0.442	0.650	0.929	0.392	3.342	3.320
11	7.54	89.98	2.48	0.225	0.346	0.570	0.754	0.392	3.351	3.780
12	10.51	85.92	3.57	0.191	0.300	0.490	0.590	0.398	3.089	4.130
13	8.94	88.19	2.87	0.187	0.264	0.410	0.497	0.410	2.658	4.890
14	10.73	85.34	3.93	0.178	0.260	0.370	0.425	0.418	2.388	5.870
15	13.93	83.13	2.94	0.173	0.252	0.335	0.372	0.426	2.150	7.100

<sup>a</sup>Represents the unitless parameters.

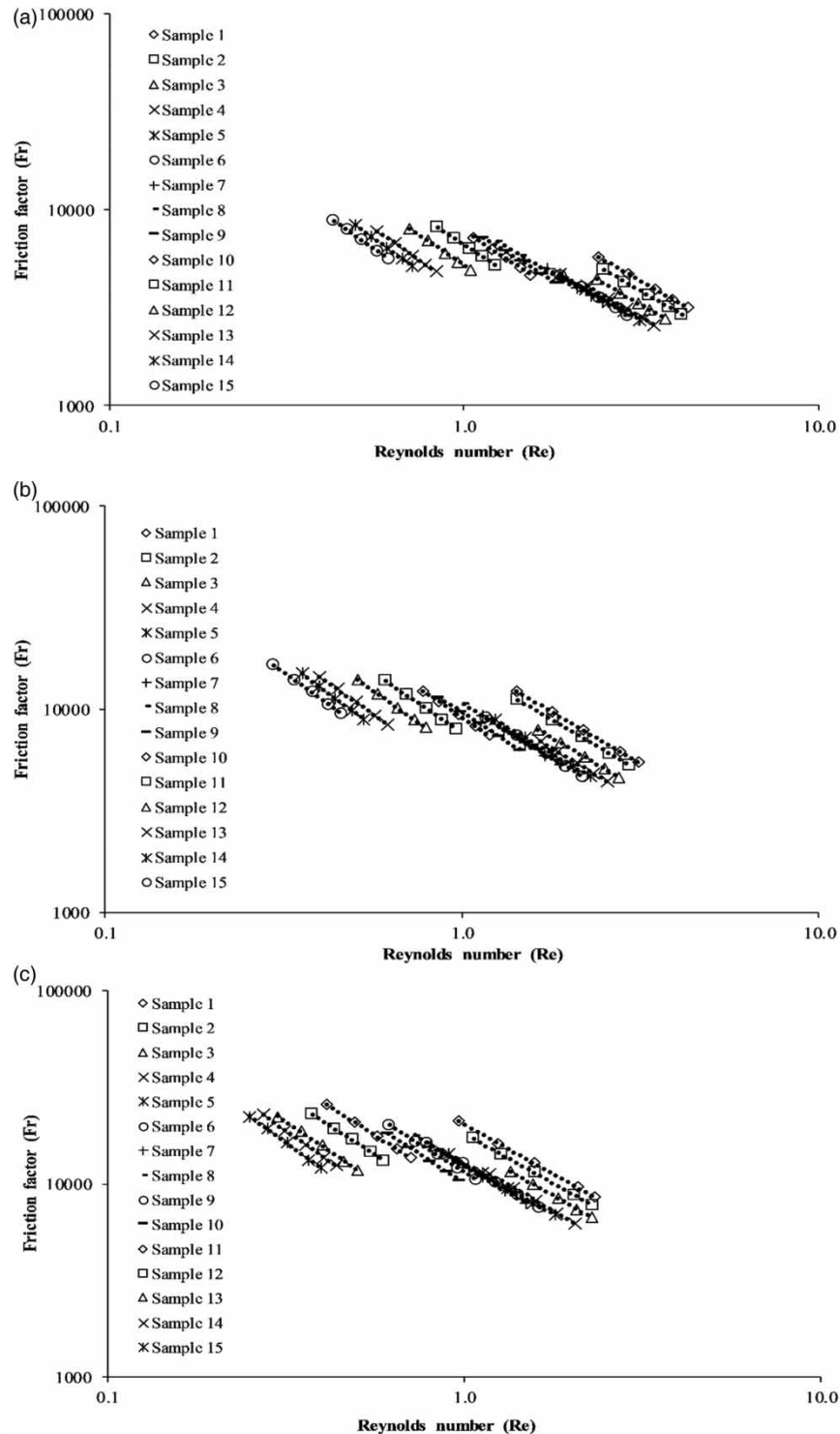
**Figure 3.** The computation of  $F_r$  and  $R_e$  values plays a crucial role in determining the flow regime. The  $F_r$  and  $R_e$  are computed as:

$$F_r = \frac{h_i * 2 * g * d_{50}}{V^2} \quad (10)$$

$$R_e = \frac{V * d_{50}}{\nu} \quad (11)$$

where,  $h_i$  = hydraulic gradient,  $V$  = average flow velocity,  $d_{50}$  = mean grain size, and  $\nu$  = fluid kinematic viscosity, and  $g$  = gravitational constant.

Figure 3 indicates straight-line variation between  $F_r$  and  $Re$  for permeameters having different diameters, which signifies that the flow is in a linear regime and thereby confirms the presence of Darcy's regime (Hellstrom & Lundstrom 2006; Alabi 2011).



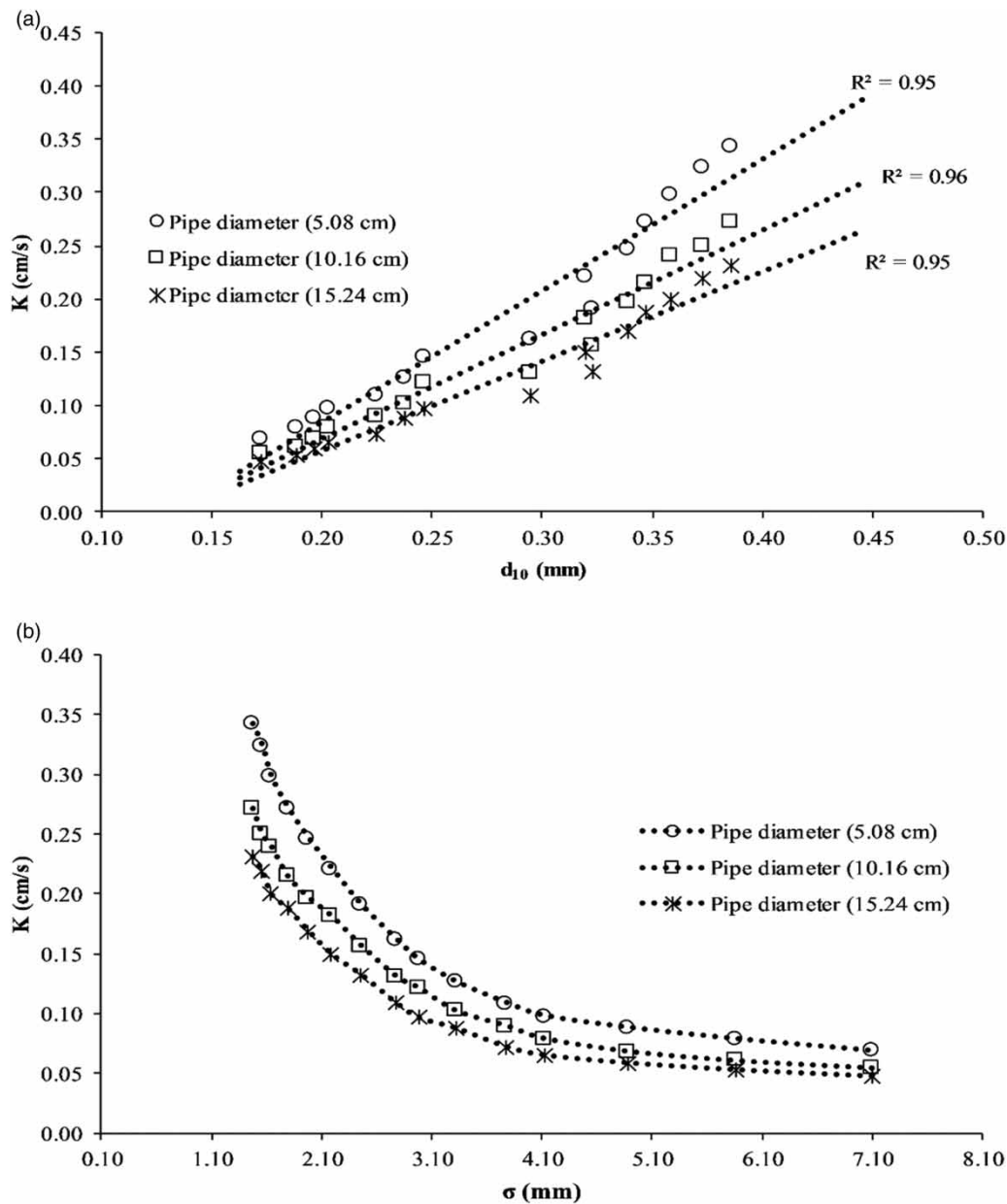
**Figure 3** |  $F_r$  and  $Re$  variation for different soil samples from (a) 5.08, (b) 10.16, and (c) 15.24 cm diameter permeameters.



### Variation of K with effective grain size and standard deviation

The hydraulic conductivity values obtained from the three permeameters were plotted against the effective grain size i.e.,  $d_{10}$  (Figure 4(a)), wherein a linear variation of K was observed with the different values of  $d_{10}$ . From this investigation, it is concluded that as the value of  $d_{10}$  increases, it results in providing more void space to the fluid to move through the interconnected voids, and thereby results in an increase in the K value which is in the line with the outcome of Pliakas & Petalas (2011). The K value varies from 0.342 to 0.068 cm/s, 0.271 to 0.054 cm/s, and 0.232 to 0.048 cm/s for 5.08, 10.16, and 15.24 cm diameter permeameter respectively.

Figure 4(b) represents the variation of K with the values of standard deviation, which vary from (1.47–7.10) for 5.08, 10.16, and 15.24 cm diameter permeameters. The hydraulic conductivity value decreases with the increase in the  $\sigma$  value. The observed value of  $\sigma$  for the soil samples is greater than 1, which represents the non-uniformity of porous media. As the value of  $\sigma$  increases the non-uniformity of porous particles increases which impart more compactness to the packed



**Figure 4** | Variations of K with (a) effective grain size; and (b) standard deviation.

media and thereby results in the decreased value of the hydraulic conductivity. As the  $K$  value decreases and tends to zero, the curves approach asymptotically towards the  $y$ -axis as shown in [Figure 4\(b\)](#).

### Hydraulic conductivity variation with $\sigma/d_{10}$

The variation of hydraulic conductivity computed from three permeameters was studied by plotting  $K$  with different values of  $\sigma/d_{10}$ . [Figure 5](#) indicates that as the  $\sigma/d_{10}$  value increases  $K$  decreases. The curve touches asymptotically the lower limiting value and is concave upward. The inferences were drawn from the study, i.e., the trend of the curve in [Figure 5](#) is in close agreement with the outcomes of [Masch & Denny \(1966\)](#) and [Pliakas & Petalas \(2011\)](#). The  $\sigma/d_{10}$  values range between 3.80 and 41.06.

Investigations on the influence of  $\sigma/d_{10}$  on the hydraulic conductivity values obtained from different diameter permeameters provide the basis for developing a novel statistical model for the computation of  $K$  of porous media.

### Statistical model development

A statistical model for the computation of hydraulic conductivity has been proposed by using the data points of 15 borehole soil samples. The grain size parameters i.e.,  $\sigma$  and  $d_{10}$  have been used for model development. The developed statistical model comprises coefficients of  $\sigma/d_{10}$  parameter of degree 0–4. The proposed model is developed using the principle of least squares analysis.

The standard form of the proposed  $K$  model is given below:

$$K = \lambda * \left[ z_0 + \sum_{i=1}^{i=4} z_i * \left( \frac{\sigma}{d_{10}} \right)^i \right] \quad \left( 3.805 \leq \frac{\sigma}{d_{10}} \leq 41.066 \right) \quad (12)$$

where,  $\lambda$  = factor that considers the compactness of the soil particles near the wall, particle roughness, wall confinement, and extent of the porous media. The values of  $\lambda$  for different diameter permeameters are given below:

Permeameter diameter (cm)	$\lambda$ values
5.08	1.23
10.16	0.96
15.24	0.82

From the observed values of  $\lambda$ , it can be seen that the  $\lambda$  value is larger for minimum permeameter diameter and smaller for maximum permeameter diameter. This observation postulates that the magnitude of  $\lambda$  becomes insignificant with the increase in the porous media extent.

The empirical constant ( $z_0$ ,  $z_1$ ,  $z_2$ ,  $z_3$ , and  $z_4$ ) values are:  $z_0 = 0.3880$ ,  $z_1 = 0.0327$ ,  $z_2 = 0.0013$ ,  $z_3 = 2.56 \times 10^{-5}$ , and  $z_4 = 2 \times 10^{-7}$ .

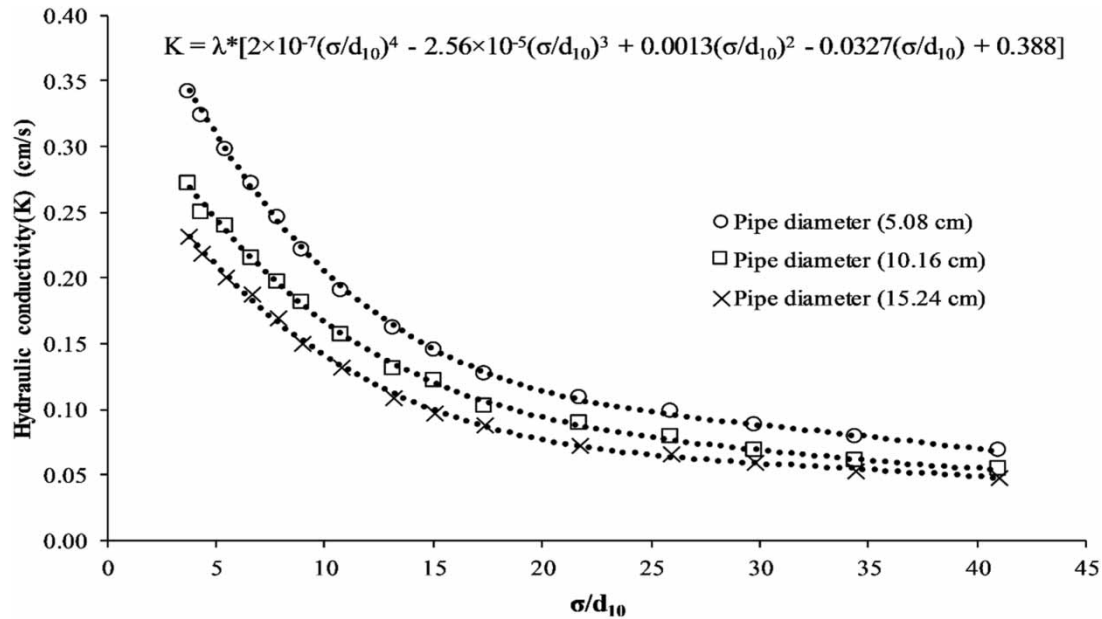
The developed statistical model for the  $K$  estimation is:

$$K = \lambda * \left[ 2 \times 10^{-7} \left( \frac{\sigma}{d_{10}} \right)^4 - 2.56 \times 10^{-5} \left( \frac{\sigma}{d_{10}} \right)^3 + 0.0013 \left( \frac{\sigma}{d_{10}} \right)^2 - 0.0327 \left( \frac{\sigma}{d_{10}} \right) + 0.388 \right] \quad (13)$$

Further, the  $\sigma/d_{10}$  and  $K$  values computed experimentally and by using the developed statistical model are given in [Table 3](#).

### Computation of $K$ using empirical equations

The hydraulic conductivity of soil samples was computed via the seven empirical equations considered in this study, as mentioned in [Table 1](#). Grain-size parameters, uniformity and sorting coefficients, and porosity values were used to compute  $K$  based on empirical equations. The value of grain size parameters is stated in [Table 2](#). A specific value of the sorting coefficient has been used in each empirical equation except the Terzaghi equation. In the Terzaghi's equation two different values of sorting coefficient i.e.,  $10.7 \times 10^{-3}$  for smooth grains and  $6.1 \times 10^{-3}$  for coarse grains have been given, therefore an average



**Figure 5** | Variations of K with  $\sigma/d_{10}$ .

**Table 3** | Experimentally measured ( $K_{\text{measured}}$ ) and developed model ( $K_{\text{model}}$ )-based K values

Sample no.	$\sigma/d_{10}$	$(K_{\text{measured}})$ (cm/s) permeameter diameter (cm)			$(K_{\text{model}})$ (cm/s) permeameter diameter (cm)		
		5.08	10.16	15.24	5.08	10.16	15.24
1	3.805	0.342	0.271	0.232	0.346	0.270	0.231
2	4.390	0.323	0.249	0.219	0.330	0.257	0.220
3	5.539	0.298	0.240	0.200	0.300	0.234	0.200
4	6.669	0.272	0.215	0.188	0.273	0.213	0.182
5	7.876	0.246	0.197	0.169	0.248	0.194	0.165
6	9.058	0.221	0.181	0.150	0.226	0.176	0.151
7	10.787	0.190	0.156	0.132	0.199	0.155	0.132
8	13.255	0.162	0.131	0.109	0.168	0.131	0.112
9	15.104	0.145	0.121	0.097	0.150	0.117	0.100
10	17.376	0.126	0.102	0.088	0.133	0.104	0.089
11	21.754	0.109	0.090	0.072	0.114	0.089	0.076
12	25.964	0.098	0.079	0.065	0.106	0.083	0.071
13	29.868	0.088	0.068	0.059	0.104	0.082	0.070
14	34.467	0.078	0.061	0.053	0.093	0.076	0.064
15	41.066	0.069	0.054	0.048	0.082	0.065	0.052

sorting coefficient value of  $8.4 \times 10^{-3}$  has been used in the Terzaghi equation (Pucko & Verbovsek 2015). The computed values of hydraulic conductivity via empirical equation are shown in Table 4.

Further, the K values determined using different diameter permeameters were compared with the values computed via the empirical equations considered in the study. The Hazen equation gives a substantially closer agreement with the measured K values for all soil samples, followed by the Kozeny–Carman equation as shown in Figure 6. The Hazen equation depends on the entire particle distribution curve, porosity, and effective particle size which makes this equation more precise in the computation of K as compared to the other equations (Carrier 2003; Ishaku *et al.* 2011). Whereas the other empirical equations

**Table 4** | Hydraulic conductivity values computed using empirical equations

Sample no.	K <sub>Hazen</sub> (cm/s)	K <sub>Slichter</sub> (cm/s)	K <sub>Terzaghi</sub> (cm/s)	K <sub>K-c</sub> (cm/s)	K <sub>Harleman et al.</sub> (cm/s)	K <sub>Chapuis et al.</sub> (cm/s)	K <sub>Naeef et al.</sub> (cm/s)
1	0.197	0.057	0.098	0.154	0.108	0.057	0.039
2	0.188	0.055	0.095	0.151	0.101	0.056	0.039
3	0.179	0.054	0.093	0.150	0.093	0.056	0.039
4	0.172	0.053	0.092	0.150	0.087	0.056	0.039
5	0.175	0.056	0.098	0.162	0.085	0.060	0.041
6	0.164	0.053	0.093	0.155	0.079	0.058	0.041
7	0.161	0.053	0.092	0.153	0.077	0.057	0.041
8	0.136	0.044	0.076	0.127	0.066	0.049	0.037
9	0.137	0.046	0.080	0.135	0.064	0.052	0.039
10	0.119	0.039	0.069	0.116	0.056	0.046	0.036
11	0.078	0.026	0.045	0.076	0.037	0.033	0.030
12	0.058	0.020	0.034	0.058	0.026	0.027	0.028
13	0.058	0.021	0.036	0.064	0.025	0.029	0.030
14	0.054	0.020	0.035	0.063	0.023	0.028	0.030
15	0.053	0.020	0.035	0.065	0.022	0.029	0.031

i.e., Slichter (1899), Terzaghi (1925), Harleman *et al.* (1963), Chapuis *et al.* (2005), and Naeef *et al.* (2017), resulted in poor agreement with the experimentally measured K values, which is consistent with the findings of Cheng & Chen (2007) and Riha *et al.* (2018) as indicated in Figure 6. The Terzaghi equation results in lower K values because of using the average value of sorting coefficient, whereas, in Chapuis *et al.* (2005) equation, the parameters namely fluid viscosity and gravitational acceleration are not considered as compared to other equations, which, may result in the underestimation of K values. Slichter (1899), Harleman *et al.* (1963), and Naeef *et al.* (2017) underestimated the K values because these empirical equations and for a well graded porous media (Rosas *et al.* 2014).

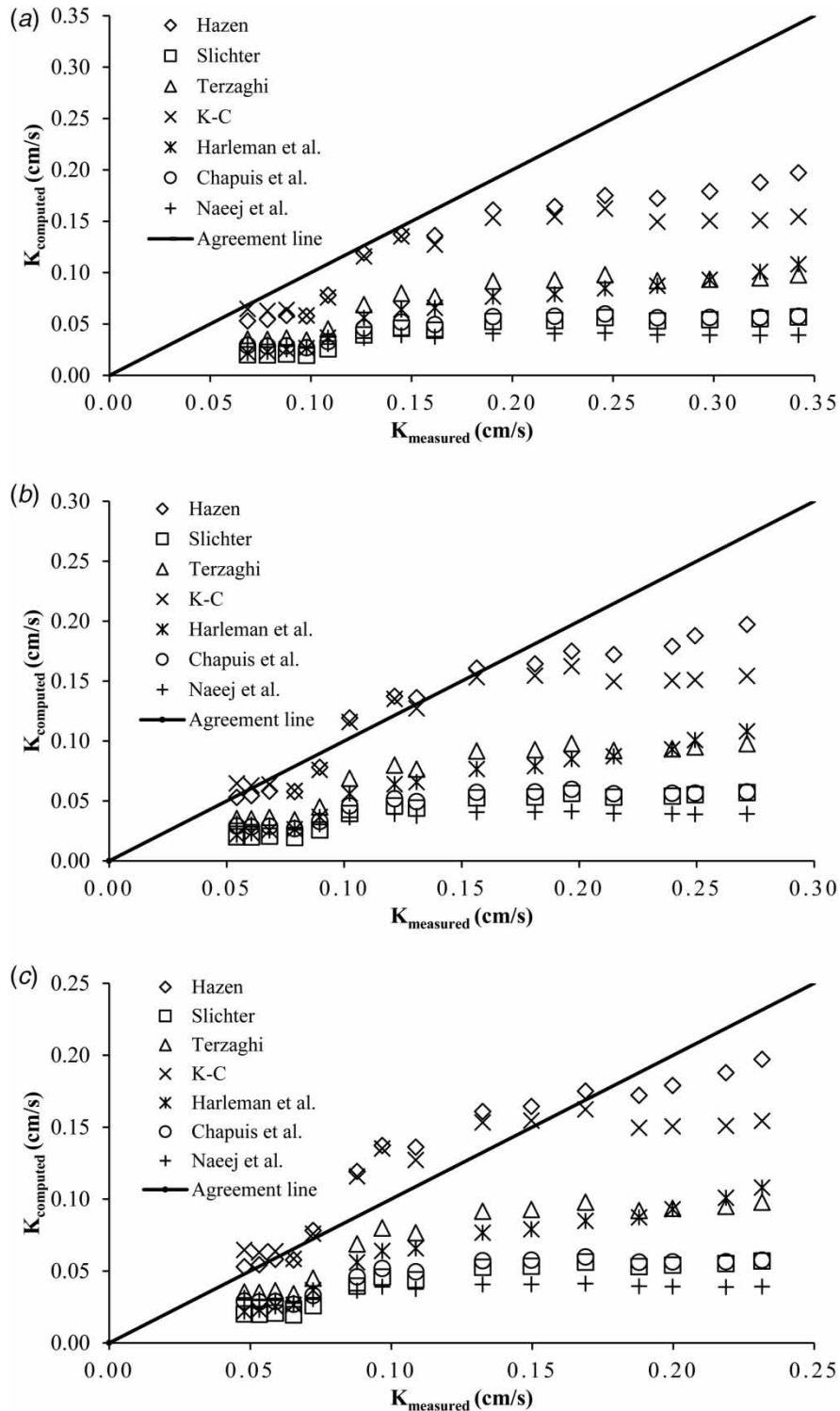
Further, the hydraulic conductivity values computed using the developed statistical model have been compared with the experimentally measured K values as shown in Figure 7. The K values obtained from the developed model show a relatively good agreement with the measured K values as compared to the empirical equations considered in the study. Figure 7 shows that the data points computed from the developed model are focused more on the agreement line as compared to Figure 6.

Further, the quantitative performance of the developed model and empirical equations were assessed using different statistical indicators for different diameter permeameters as given in Table 5.

The values of  $I_a$  and  $R^2$  vary from 0 to 1, and  $S_i$ , MAE, RMSE, and BIAS from 0 to  $\infty$  (Naeef *et al.* 2017). BIAS may result in negative values which indicates the lower values of computed parameters as compared to the measured parameter. Values closer to 1 for  $I_a$  and  $R^2$ , and lower values of  $S_i$ , MAE, RMSE, and BIAS indicate a better correlation between measured and computed parameters. Among the seven empirical equations considered in the study, the Hazen equation results in the lower values of  $S_i$ , MAE, RMSE, and BIAS, and values are close to 1 for  $I_a$  and  $R^2$  for different diameter permeameters as shown in Table 5. This postulates that the Hazen equation performs relatively well in K estimation as compared to the other equations. The values of statistical indicators i.e.,  $S_i$ , MAE, RMSE, and BIAS are 0.045, 0.007, 0.008, & 0.007 for 5.08 cm, 0.047, 0.005, 0.007, and 0.002 for 10.16 cm, and 0.039, 0.004, 0.005, and 0.002 for 15.24 cm diameter permeameters respectively are significantly reduced for the developed statistical model, whereas the  $I_a$  and  $R^2$  values are 0.984 and 0.982, 0.985 and 0.972, and 0.978 and 0.953 for 5.08, 10.16, and 15.24 cm diameter permeameters respectively. The quantitative evaluation via the statistical indicators establishes the efficacy of the developed statistical model in computing the K of porous media.

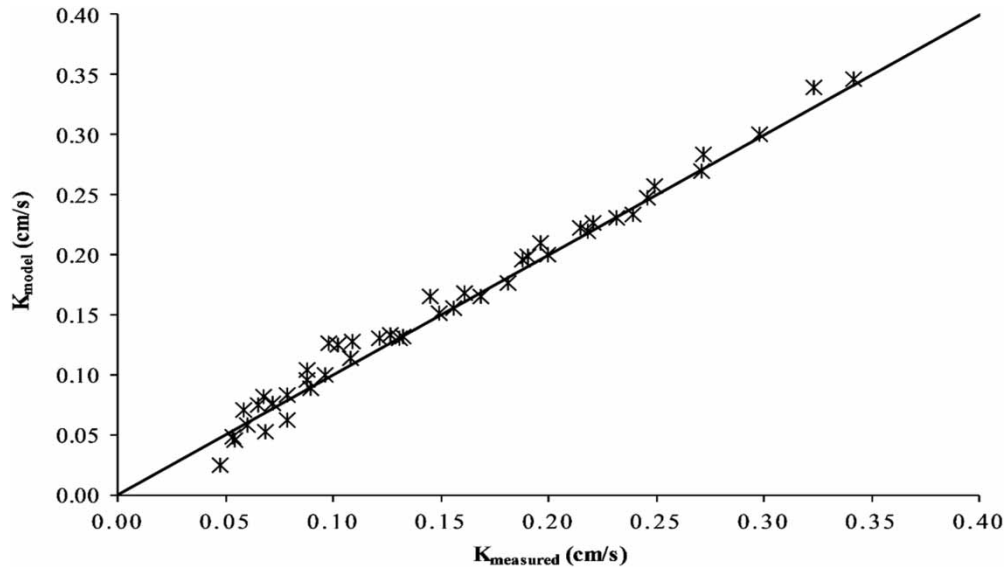
### Validation of the developed model

For the validation of the developed model, the data points corresponding to the remaining 12 soil samples have been used. Initially, the standard deviation and effective grain size for these samples have been determined. The developed model is composed of the parameter ' $\sigma/d_{10}$ ' for computing the K value. Therefore, the values of  $\sigma/d_{10}$  for these soil samples have been determined as shown in Table 6.



**Figure 6** | Comparison of measured and computed  $K$  for (a) 5.08, (b) 10.16, and (c) 15.24 cm diameter permeameters.

By using the  $\sigma/d_{10}$  value for different soil samples and  $\lambda$  value for different diameter permeameters, the  $K$  value has been determined using the developed model. Further, for validation, the predicted values of  $K$  have been compared with the values measured using the permeameters for these 12 soil samples as shown in Figure 8.



**Figure 7** | Comparison of measured and developed model-based K values.

**Table 5** | Statistical indicators for developed model and empirical equations

Permeameter diameter	Statistical parameters	Developed model	Hazen	Slichter	Terzaghi	Kozeny-Carman	Harleman et al.	Chapuis et al.	Naeij et al.
5.08 cm	BIAS	0.007	-0.056	-0.143	-0.113	-0.066	-0.121	-0.138	-0.148
	Si	0.045	0.390	0.883	0.717	0.483	0.740	0.865	0.932
	R <sup>2</sup>	0.982	0.874	0.794	0.783	0.824	0.802	0.722	0.593
	I <sub>a</sub>	0.984	0.951	0.881	0.891	0.915	0.879	0.883	0.861
	MAE	0.007	0.056	0.143	0.113	0.066	0.121	0.138	0.148
	RMSE	0.008	0.072	0.163	0.132	0.089	0.136	0.160	0.172
10.16 cm	BIAS	0.002	-0.019	-0.107	-0.076	-0.029	-0.084	-0.101	-0.112
	Si	0.047	0.227	0.822	0.616	0.343	0.643	0.800	0.884
	R <sup>2</sup>	0.972	0.893	0.819	0.808	0.842	0.834	0.749	0.621
	I <sub>a</sub>	0.985	0.946	0.904	0.875	0.938	0.896	0.889	0.863
	MAE	0.005	0.025	0.107	0.076	0.034	0.084	0.101	0.112
	RMSE	0.007	0.034	0.121	0.091	0.051	0.095	0.118	0.130
15.24 cm	BIAS	0.002	0.003	-0.084	-0.054	-0.007	-0.062	-0.079	-0.089
	Si	0.039	0.177	0.776	0.537	0.276	0.565	0.751	0.849
	R <sup>2</sup>	0.953	0.879	0.802	0.791	0.846	0.865	0.731	0.604
	I <sub>a</sub>	0.978	0.953	0.902	0.885	0.938	0.913	0.876	0.862
	MAE	0.004	0.018	0.084	0.054	0.026	0.062	0.079	0.089
	RMSE	0.005	0.022	0.097	0.067	0.035	0.071	0.094	0.106

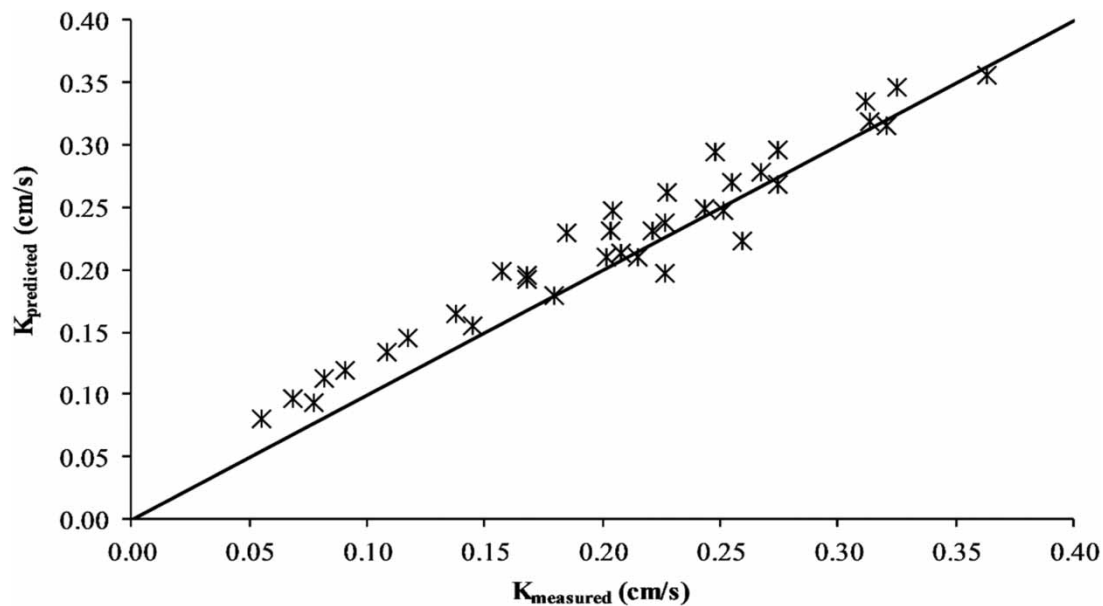
From Figure 8 it has been observed that the hydraulic conductivity values predicted using the developed model shows fairly good agreement with the measured K values. During validation of the developed model the values of statistical indicators i.e., R<sup>2</sup>, I<sub>a</sub>, BIAS, S<sub>i</sub>, MAE, and RMSE are 0.942, 0.894, 0.044, 0.052, 0.025, and 0.018 respectively, which substantiate the performance of the developed model in computing the K value of porous media.

## CONCLUSIONS

The present study is devoted to establishing a statistical model for the computation of the hydraulic conductivity based on the grain size parameters i.e.,  $d_{10}$  and  $\sigma$ . The developed model includes a factor ' $\lambda$ ' which incorporates the porous media

**Table 6** | Hydraulic conductivity predicted using the developed model

Sample no.	Gravel (%)	Sand (%)	Silt (%)	$\sigma/d_{10}$	(K <sub>predicted</sub> ) (cm/s)			(K <sub>measured</sub> ) (cm/s)		
					Permeameter diameter (cm)			Permeameter diameter (cm)		
					5.08	10.16	15.24	5.08	10.16	15.24
1	22.86	74.46	2.68	3.519	0.355	0.277	0.236	0.364	0.268	0.227
2	20.37	75.68	3.95	3.831	0.345	0.270	0.230	0.325	0.255	0.222
3	21.65	76.49	1.86	4.800	0.319	0.249	0.212	0.314	0.244	0.208
4	18.62	75.94	5.44	5.714	0.295	0.231	0.197	0.275	0.204	0.227
5	16.76	79.74	3.50	4.930	0.315	0.246	0.210	0.321	0.252	0.202
6	14.81	82.85	2.34	4.215	0.335	0.261	0.223	0.312	0.228	0.260
7	15.69	81.76	2.55	5.818	0.293	0.229	0.195	0.248	0.185	0.168
8	13.14	84.26	2.60	6.920	0.268	0.209	0.178	0.275	0.215	0.180
9	12.18	86.79	1.03	7.982	0.246	0.192	0.164	0.205	0.168	0.138
10	11.76	85.46	2.78	10.792	0.199	0.155	0.132	0.158	0.145	0.109
11	9.86	88.57	1.57	15.850	0.144	0.112	0.096	0.118	0.082	0.069
12	10.39	86.75	2.86	20.160	0.119	0.093	0.080	0.091	0.078	0.056

**Figure 8** | Comparison of predicted K with the measured values.

compactness, particle roughness, and extent of the porous media. The observed values of  $\lambda$  indicate that the magnitude of  $\lambda$  becomes insignificant with the increase in the porous media extent. The influence of  $d_{10}$  and  $\sigma$  on the K of borehole soil samples elucidate that, with the increase in the  $d_{10}$  grain size, the K of soil samples increases and decreases with the increase in the  $\sigma$  value. The  $F_r$  and  $R_e$  variations indicate the existence of flow in Darcy's regime. The comparative evaluation of seven empirical equations indicates that the Hazen equation shows a relatively good agreement with the measured K values. The quantitative evaluation using the statistical indicators substantiates the efficacy of the developed statistical model in the computation of K of porous media. The BIAS,  $S_i$ ,  $R^2$ ,  $I_a$ , MAE, and RMSE for the developed model are 0.007, 0.045, 0.982, 0.984, 0.007, and 0.008 for 5.08 cm, 0.002, 0.047, 0.972, 0.985, 0.005, and 0.007 for 10.16 cm, 0.002, 0.039, 0.953, 0.978, 0.004, and 0.005 for 15.26 cm diameter permeameters respectively. The study also validates the performance of the developed model in



computing the K for the independent data set. The developed model provides an effective tool to compute the aquifer yield, groundwater recharge, and filter design with precise accuracy.

## DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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