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Performance of rectangular labyrinth weir – an experimental and numerical study

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ABSTRACT

Labyrinth weirs are commonly used to increase the capacity of existing spillways and provide more efficient spillways for new dams due to their high specific discharge capacity compared to the linear weir. In the present study, an experimental and numerical investigation was conducted to improve the rectangular labyrinth weir performance. In this context, four configurations were tested to evaluate the influence of the entrance shape and alveoli width on its discharge capacity. The experimental models, three models of rectangular labyrinth weir with a rounded entrance and one with a flat entrance, were tested in rectangular channel conditions for inlet width to outlet width ratios (a/b) equal to 0.67, 1 and 1.5. The results indicate that the rounded entrance increases the weir efficiency by up to 5%. A ratio a/b equal to 1.5 leads to an 8 and 18% increase in the discharge capacity compared to a/b ratio equal to 1 and 0.67, respectively. In addition, a numerical simulation was conducted using the open-source Computational Fluid Dynamics (CFD) OpenFOAM to analyze and provide more information about the flow behavior over the tested models. A comparison between the experimental and numerical discharge coefficient was performed and good agreement was found (mean absolute relative error of 4–6%).

Key words: control structures, discharge estimation, rectangular labyrinth weir

HIGHLIGHTS

- This paper presents the efficiencies of rectangular labyrinth weirs with flat and rounded entrances for channel applications.
- The rectangular labyrinth weir efficiency is mainly dependent on its inlet alveoli capacity.
- Both numerical and experimental models provided close results.
- An InterFoam solver produced more information about the effect of inlet shape and the alveoli widths.

GRAPHICAL ABSTRACT

Dams are generally affected by two main problems, the safety issue due to the limited capacity of the spillway to evacuate the extreme floods and storage losses due to the silting. In order to deal with these issues, dam operators commonly choose the labyrinth weirs to address storage and evacuation deficiencies.
The experimental and numerical results of this study have shown that the rounded rectangular labyrinth, with L/W = 4 and a/b = 1.5, is capable of increasing the discharge capacity by up to two to three times compared with a linear weir having the same width and upstream head, mainly for low and moderate relative upstream heads.

1. INTRODUCTION

The labyrinth weir is a non-linear flood control structure providing an increase in the unit discharge capacity relative to linear weirs (Figure 1). The use of non-linear shape in plan view enables the labyrinth weir crest length to be increased within the existing spillway width. This option leads to an augmentation in the discharge capacity of the labyrinth weir by up to three to four times compared with a linear weir having the same width and upstream head (Tullis *et al.* 1995). Consequently, the adoption of a labyrinth weir can effectively reduce a direct and/or indirect cost of the projects of rehabilitation or construction of new dams. Contrary to a labyrinth weir, a linear weir requires a larger width or low crest elevation to increase the discharge efficiency due to its low specific discharge capacity. For example, the specific discharge for a standard Creager type is close to 2.15 $h^{1.5}$, where h is the upstream head in meters (Hydrocoop 2013). A greater width of the crest of the spillway and downstream channel result in an increase in the volume of materials required for construction, and can directly affect the total cost of the structure. The construction of the crest of the dam with a lower level can reduce the normal elevation of the reservoir and thus the storage capacity of the dam, which may have an indirect negative impact on the project cost. As a result, the labyrinth weir is well suited for a dam or in a natural or artificial channel application to increase at low cost the spillway evacuation and/or storage capacity. The main geometric parameters of a labyrinth weir are the developed crest length L, the total width $W_{t,}$ the cycle width w, the weir height P, the total upstream $H_{t,}$ cycle number N, and the sidewall angle α. The first reported application of labyrinth weirs was conducted by Murphy (1909) to provide a high discharge capacity for the reinforced-concrete spillway of the Keno Canal, in the USA. It is highlighted that a linear weir of equivalent discharge

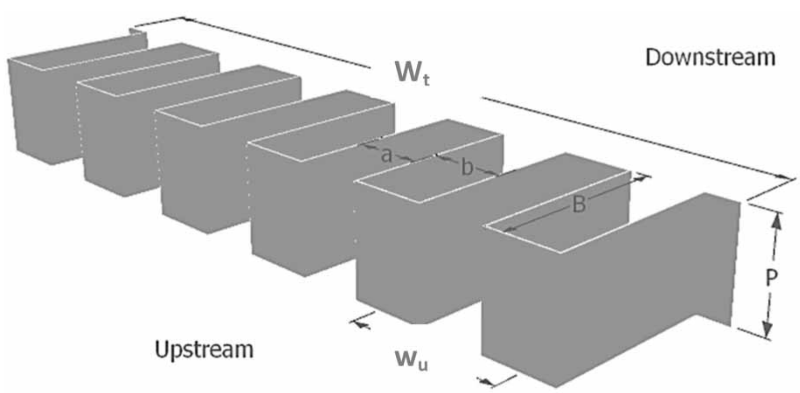

Figure 1 | Main geometrical parameters of a rectangular labyrinth weir.

capacity would need to be three times the length of the adopted labyrinth weir. However, the first study of the labyrinth weir performance was performed by Gentilini (1940). He tested a labyrinth weir designed with triangular shape in plan view and compared it with a linear weir capacity. Nevertheless, the significant development in the understanding of the effects of the labyrinth weir geometry on discharge capacity was conducted by Taylor (1968) and Hay & Taylor (1970), specifically for trapezoidal and triangular labyrinths. Later, an extensive investigation was mainly conducted on the trapezoidal labyrinth form by many researchers (e.g., Darvas 1971; Houston 1983; Hinchliff & Houston 1984; Lux & Hinchliff 1985; Tullis *et al.* 1995, 2007; Falvey 2003; Lopes *et al.* 2009; Crookston & Tullis 2012; Dabling *et al.* 2013). It was found in those works that the performance of trapezoidal labyrinth weirs is mainly governed by the total relative head ratio H_t/P, the developed crest length ratio, L/W, or the sidewall angle, α, the cycle width ratio w/P, and the crest shape.

Recently, a numerical simulation method has been widely used to further analyze and design the labyrinth weir. This method has been applied on the trapezoidal labyrinth form by Savage *et al.* (2016) and it provided a good result compared to the experimental data (error of 3%). Carrillo *et al.* (2019) carried out a numerical simulation to analyze the trapezoidal labyrinth efficiency for free and submerged conditions. Ghaderi *et al.* (2020) performed numerical and experimental tests to analyze the energy dissipation and hydraulics of flow over trapezoidal and triangular labyrinth shape. They conclude that the numerical model is suitable to accurately predict the flow behavior of the labyrinth weir with a relative error of 3.05%. Safarzadeh & Noroozi (2017) numerically investigated the flow features over the rectangular labyrinth and designed Piano Key Weir with trapezoidal shape in plan view.

1.1. Rectangular labyrinth weir performance

The rectangular labyrinth has a simple geometry, combines high performance and a low construction cost. However, far too little attention has been paid to the study of the performance of labyrinth weir having a rectangular shape in plan view. Kabiri-Samani *et al.* (2013) studied a rectangular labyrinth weir designed with a flat entrance. The influence of the free and submerged flow conditions on the discharge capacity of the rectangular labyrinth weir has been investigated by Azimi & Seyed Hakim (2019). They indicated that the rectangular labyrinth weir is more efficient than the sharp-crested weir and it is less sensitive to submergence and it behaves ideally within the upstream relative head (h_0) range of $0.3 \leq h_0/P \leq 0.4$.

Lempérière & Ouamane (2003), Ouamane & Ben Said (2010) and (Ben Said & Ouamane 2011, 2018) studied several physical models of rectangular labyrinth weirs designed with entrances having a rounded shape and found that the rectangular labyrinth weir is more efficient than the trapezoidal weir mainly for moderate discharge. The rectangular labyrinth shape was found to provide the highest discharge capacity compared to the trapezoidal shape for L/W ratio equal to 4 and 5. Using a rectangular shape enables a reduction in the footprint of the weir, which reduces the construction cost and allows it to be installed in a limited area. In addition, through choosing a rectangular shape, the free height walls can be decreased by the partial filling of the downstream alveoli with ordinary concrete, allowing the steel-reinforced concrete to be reduced, which can result in a reduction in the amount of the construction cost without compromising its hydraulic efficiency (Ouamane & Ben Said 2010; Ben Said & Ouamane 2011). Figure 1 shows the main geometric parameters of the rectangular labyrinth weir including the total width of the weir W_t, the cycle width w_u, the weir height P, the inlet alveoli width a, the outlet alveoli width b, and the lateral crest length B. In addition, the radius r of the rounded entrance was considered for the rectangular labyrinth having rounded shape (see Figure 4).

The main objective of this study was to analyze the rectangular labyrinth weir discharge efficiency using experimental and numerical modeling. For that, the sensitivity of the rectangular labyrinth weir to the entrance shape and alveoli width was measured on four models of rectangular labyrinth weirs. Firstly, two models with the same a/b ratio = 1, were tested to investigate the influence of the entrance shape on the rectangular labyrinth. Then, the optimal model from the previous comparison was chosen as the reference model and compared to the two other models with ratios a/b = 0.67 and 1.5, to investigate the influence of the variation of the alveoli width on the rectangular labyrinth.

1.1.1. Labyrinth weir discharge capacity

The head discharge relationship for a linear sharp-crested weir is generally used to quantify the labyrinth weir capacity:

$$Q = C_w \sqrt{2g} W H^{\frac{3}{2}} \qquad (1)$$

where

Q: discharge over weir (m^3/s)
C_w: discharge coefficient
g: gravity acceleration (m/s^2)
W: weir width (m)
H: total upstream head (m).

The height, the crest shape, and the crest length are the main parameters influencing the linear weir capacity (Darvas 1971). However, the principal geometric parameters affecting the performance of labyrinth weirs are: the total weir crest length L, the weir width W, the weir height P and the inlet and outlet alveoli widths a and b.

In this study, the discharge coefficient C_w is estimated by using the experimental H ($H = h + U^2/2\,g$) and Q data. Based on dimensional analysis, the discharge coefficient for rectangular labyrinth weir can be written as:

$$C_w = f\left(\frac{H}{P}, \frac{L}{W}, \frac{a}{b}, \frac{W}{W_c}\right) \quad (2)$$

where

H: total upstream head
h: piezometric head
b: outlet alveoli width
W_c: channel width
a: inlet alveoli width
P: weir height
L: developed crest length
W: the weir width.

2. METHODOLOGY

2.1. Experimental method

The experimental tests on physical models of the rectangular labyrinth weir were conducted at the laboratory of Hydraulic Planning and Environment of Biskra University in a straight rectangular channel 12 m long, 1 m wide and 1 m deep (Figure 2). The experimental system was equipped with two pumps delivering up to 180 l/s into an upstream stilling basin containing a perforated wall to ensure uniform approach flow conditions. A portion of the sidewalls of the channel was designed with glass to allow the observation of flow behavior (Figure 3). An electromagnetic and ultrasonic flowmeter for measuring discharge and a series of ultrasonic level sensors to measure the flow depth were the instruments of measurement used in this work. The assembly operates within a closed circuit. It begins when the water exits the storage tank to enter the flume through the stilling basin. Then, for each discharge rate, the inflows were allowed to cross the crest of weir installed on the flume bottom for five minutes before recording the data. Finally, the evacuated water flows over the downstream bed of the flume back to the storage tank. Four rectangular labyrinth weir models with six cycles were tested (see Figure 4) for $0.67 \leq a/b \leq 1.5$ and carefully inserted and leveled. All labyrinth models with a weir height of $P = 15$ cm were fabricated using 0.2 cm thick metal sheeting (Table 1). For all tests conducted in this study, free flow conditions were adopted, and around 15 discharges between 30 and 150 l/s were considered for each model. The tests were repeated three times to check the accuracy of the measurement, which gives a total of 180 runs.

2.2. Numerical method

2.2.1. Governing equations

Numerical modeling of flow over rectangular labyrinth weir was performed using OpenFOAM software, which is an open-source Computational Fluid Dynamics (CFD). The InterFoam solver, distributed with OpenFOAM for treating two-phase flow cases, was adopted in this study. This solver uses the Finite Volume Method (FVM) for solving the Reynolds-Averaged

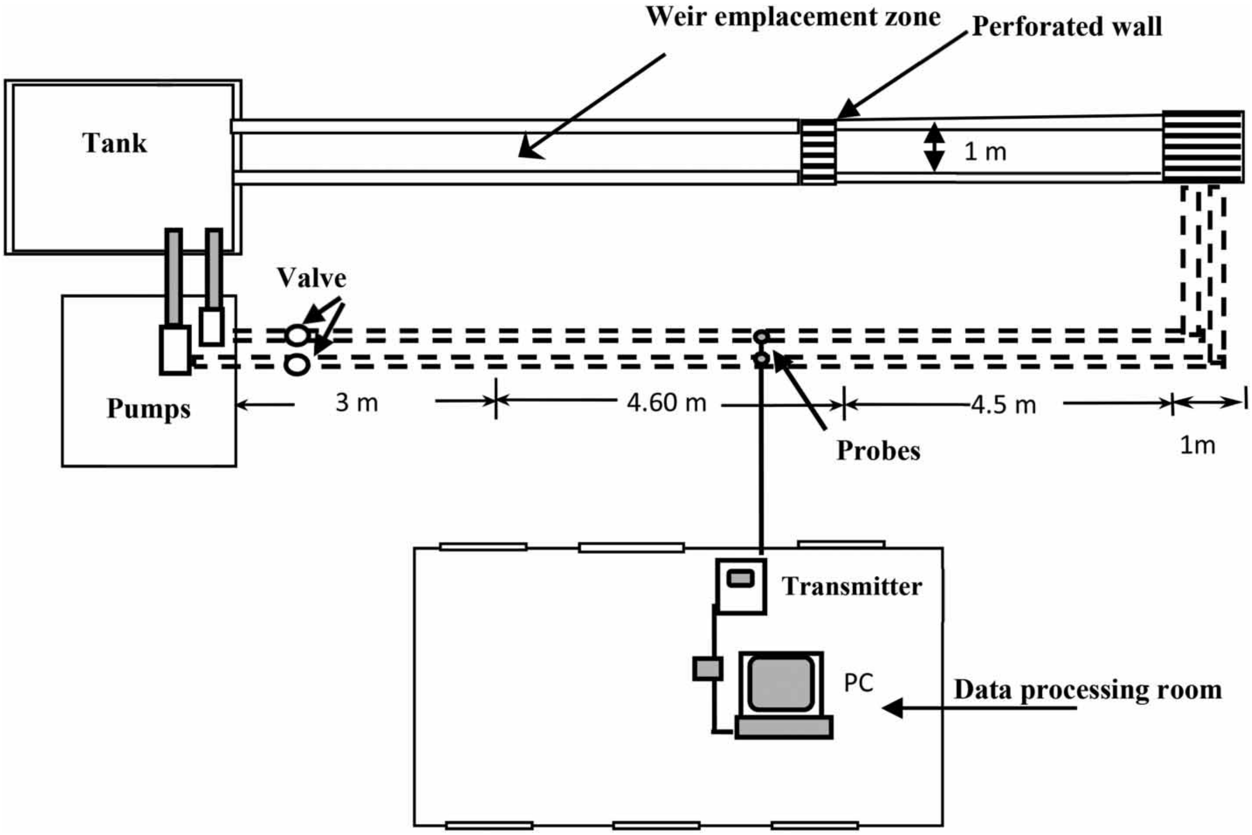

Figure 2 | View of the experimental flume.

Figure 3 | Channel flow over labyrinth weir with rounded entrance.

Navier–Stokes (RANS) equations, where the pressure implicit splitting of operators (PISO) algorithm is used to define the flow pressure p and velocity fields U (Issa 1986). InterFoam is able to capture the interface between two fluids by determining the volume fraction α. The value of α can have a value of 0 when cell contains only air, or 1 when cell is fully occupied with water. An intermediate value represents the cell where the free surface should be located. The standard k-ε turbulence model is one among others used with InterFoam for resolving the turbulence parameters and was selected here for its acceptable

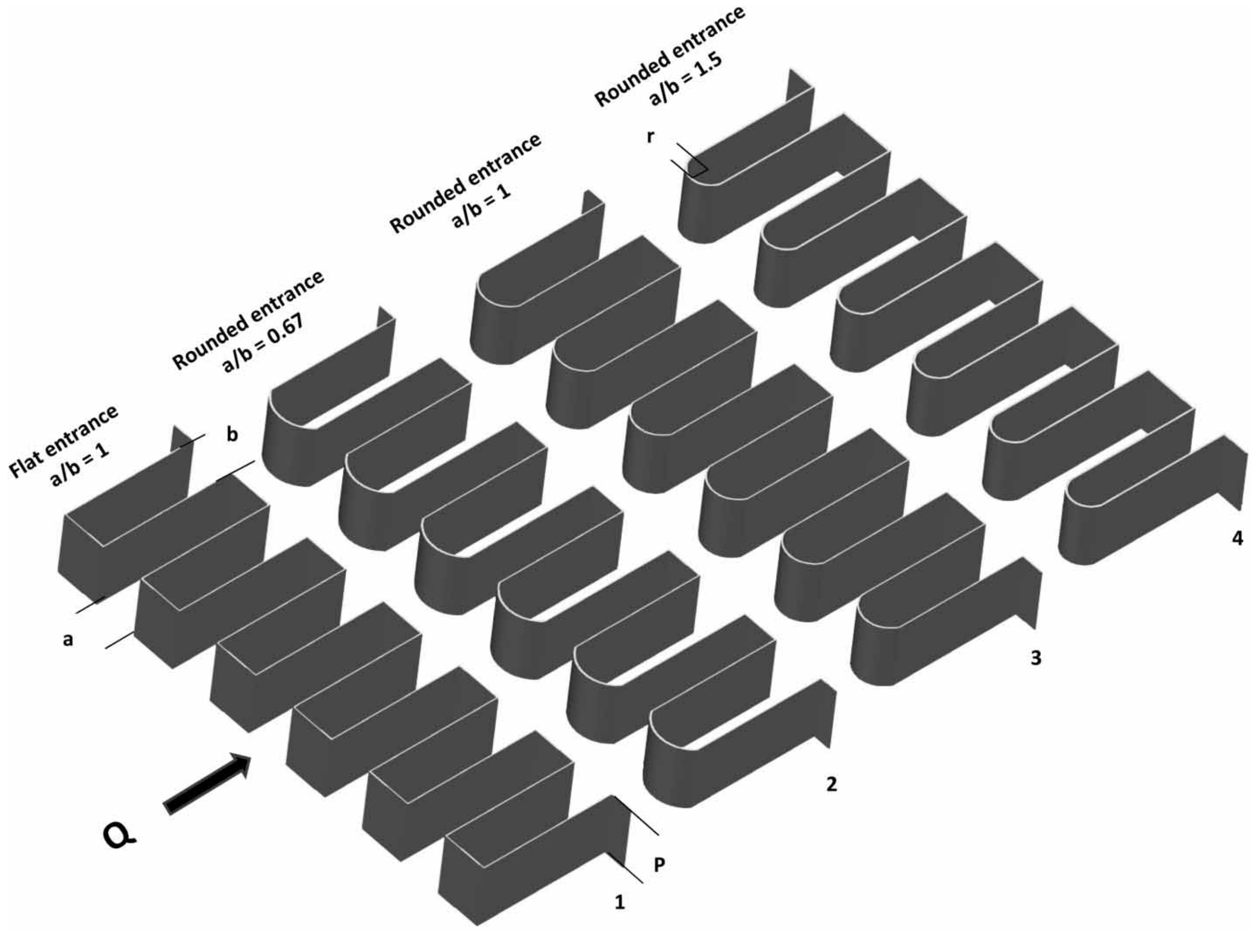

Figure 4 | Schematic of the labyrinth weir geometries considered in the analysis of the influence of the inlet entrance and alveoli width on discharge efficiency (see Table 1 for further details).

Table 1 | Characteristics of models

Model	N°	Cycle number	L (cm)	W_t (cm)	P (cm)	B (cm)	W_u (cm)	a (cm)	b (cm)	r (cm)	L/W_t	W/P	a/b	B/P
Flat entrance	1	6	351	90	15	25	58.5	7.5	7.5	–	3.9	1	1	1.67
Rounded entrance	2	6	353	90.3	15	25	58.83	6	9	4.5	3.91	1	0.67	1.67
Rounded entrance	3	6	355	90	15	25	58.5	7.5	7.5	3.75	3.9	1	1	1.67
Rounded entrance	4	6	355	90	15	25	59.16	9	6	3	3.91	1	1.5	1.67

ability for simulating free surface flow (Beg *et al.* 2017; Duguay *et al.* 2017). This turbulence model has two equations systems, in which the transport equations should be solved for estimating the turbulent kinetic energy (k) and the dissipation rate of energy (ε). The general form of the equations for continuity and momentum conservation for incompressible flows are shown as Equations (3) and (4) (Moukalled *et al.* 2015):

$$\nabla.\mathbf{U} = 0 \tag{3}$$

$$\frac{\partial}{\partial \boldsymbol{t}}[\boldsymbol{\rho}\mathbf{U}] + \nabla.\{\boldsymbol{\rho}\mathbf{U}\mathbf{U}\} = -\nabla \boldsymbol{p} + [\nabla.\boldsymbol{\tau}] + \boldsymbol{f_b} \tag{4}$$

where U is the flow velocity, ρ is the density, t is the time, p is the pressure, τ is the shear-rate tensor, f_b is the body forces.

2.2.2. Computational mesh and boundary conditions

The geometry of the tested models of the rectangular labyrinth weir was generated by using the open-source parametric 3D modeler freeCAD. Then, the blockMesh and snappyHexMesh utilities provided in OpenFOAM have been used to discretize the computational domain. First, a background hex mesh defining the computational domain was created by the blockMesh utility. Then, the snappyHexMesh utility was used to fit the geometry based on the mesh previously generated by blockMesh. The upstream and downstream parts of the numerical channel were approximately 1.8 m (12P) and 0.35 m, respectively. However, the height of the numerical domain was a function of the upstream head. The applied boundary conditions on the model were the inlet at the entrance of the channel, the atmosphere at the top of the channel, the walls for sidewalls, the bottom and the weir, and the outlet for the end of the channel. The boundary conditions are similar for all cases with the exception that the discharge (Q) is defined at the inlet.

A variable HeightFlowRateInletVelocity boundary condition was considered for U at the entrance of the domain. At the top of the channel a pressure InletOutletVelocity condition for U was defined. At the walls, the boundary conditions were applied as no-slip and a zero gradient condition was used at the outlet. A structured mesh was adopted to discretize the computational domain of simulation. In order to reduce the required time of computation for the simulations, a grid refinement was used where the weir is located to accurately identify the weir geometry and the flow behavior along the different part of the weir.

To investigate the influence of the mesh resolution, the Grid Convergence Index (GCI) method was used to estimate the accuracy of the numerical results. Consequently, three meshes with different cell sizes, 11.2, 9.4 and 7.8 mm, corresponding to 324,305, 554,924 and 953,255 cells, respectively, were used. A refinement factor between the coarse and fine gride, r, equal to 1.2 was considered, which is greater than 10% as recommended by Roache (1997) and adopted from previous studies (e.g., Aydin & Emin Emiroglu 2013; Aydin & Emin Emiroglu 2016).

The GCI was computed for the total upstream head H provided by the rectangular labyrinth weir with flat entrance using three rates of discharge equal to 37.1, 54.5 and 78.45 l/s.

Following this method which is outlined by Celik *et al.* (2008) based on Richardson extrapolation method, the representative cell size was defined as:

$$\boldsymbol{\lambda} = \left[\frac{1}{\boldsymbol{N}}\sum_{\boldsymbol{i}=1}^{\boldsymbol{N}}(\boldsymbol{\Delta V_i})\right]^{\frac{1}{3}} \tag{5}$$

where ΔV_i is volume of ith cell and N is number of cells. The apparent order P for the three-grid solution can be calculated by solving Equation (5):

$$\boldsymbol{p} = \frac{1}{\boldsymbol{lnr}_{21}}\boldsymbol{ln}|\boldsymbol{\varepsilon}_{21} - \boldsymbol{\varepsilon}_{32}| \tag{6}$$

where the value of the refinement factor $\mathbf{r_{21}} = \boldsymbol{\lambda}_2/\boldsymbol{\lambda}_1$ and $\mathbf{r_{32}} = \boldsymbol{\lambda}_3/\boldsymbol{\lambda}_2$. The ε is the error between two adjacent meshes, where $\boldsymbol{\varepsilon}_{21} = \boldsymbol{f}_2 - \boldsymbol{f}_1$ and $\boldsymbol{\varepsilon}_{32} = \boldsymbol{f}_3 - \boldsymbol{f}_2$. For this simulation, an average apparent order P_{ave} equal to 6.39 was used to calculate the GCI. The GCI was estimated using the following relation:

$$\boldsymbol{GCI} = \frac{1.25\,|\boldsymbol{E}|}{\boldsymbol{r^p} - 1} \tag{7}$$

where E is the approximate relative error between two meshes computed as:

$$\boldsymbol{E}_{21} = \frac{\boldsymbol{f}_1 - \boldsymbol{f}_2}{\boldsymbol{f}_1} \quad \text{and} \quad \boldsymbol{E}_{32} = \frac{\boldsymbol{f}_2 - \boldsymbol{f}_3}{\boldsymbol{f}_2} \tag{8}$$

f_1, f_2 and f_3 are the key variables obtained from the numerical simulation on the three grids, where f is the total upstream head provided by the tested labyrinth weir.

Table 2 shows the GCI estimations and the relative change between two adjacent meshes are presented. GCI_{21} and GCI_{32} are the relative change from medium to coarse and from fine to medium mesh, respectively. Regarding these obtained results, the values of $GCI_{32}/r^P GCI_{21}$ are close to 1, indicating that the results of the numerical model are in the asymptotic range. Consequently, a grid mesh with 554,924 cells was selected (Figure 5).

3. RESULTS

3.1. Experimental results of the discharge coefficient

3.1.1. Entrance effect

Two models of rectangular labyrinth weirs were considered to test the influence of the entrance shape on its discharge capacity. These models have the same ratio L/W = 4 and the same alveoli width a/b = 1, with the exception of the entrance shape. The first model has a flat entrance and the second was designed with a rounded entrance. The obtained results have shown that the rectangular labyrinth weir with rounded entrance is slightly more efficient than the labyrinth with flat entrance. It provides a relatively constant gain in discharge efficiency around 5% (Figure 6(a)).

3.1.2. Alveoli width effect a/b

In this investigation, the model of the rectangular labyrinth weir with a round entrance and symmetric alveoli width (a/b = 1) was chosen as the reference model to evaluate the influence of the alveoli width on the labyrinth weir efficiency. For that, two models of a rectangular labyrinth weir with a rounded entrance and varied alveoli width ratio a/b = 0.67 and 1.5, were compared to the refence model. The analysis of the experimental results showed that the rectangular labyrinth with ratio a/b = 1.5 gave the best discharge capacity mainly for moderate heads ratio (H_t/P). The model with a/b = 1.5 led to an increase by up to 8% compared to a ratio equal to 1 and 18% when the model has an a/b ratio = 0.67 as shown in Figure 6(b).

Table 2 | Grid index convergence estimation

Model	Discharge Q (l/s)	Total head H_t (m)								
		f_1	f_2	f_3	P_{ave}	E_{21}	E_{32}	GCI_{21}	GCI_{32}	$GCI_{32}/r^P GCI_{21}$
Rectangular labyrinth with flat entrance	37.1	0.0359	0.0358	0.0357	6.39	1.9973	1.9973	1.2244	1.2244	0.9973
	54.4	0.0558	0.0557	0.0561		0.1718	0.6886	0.1053	0.4221	1.0017
	78.45	0.0815	0.0814	0.0821		0.1151	0.8071	0.0706	0.4947	1.0012

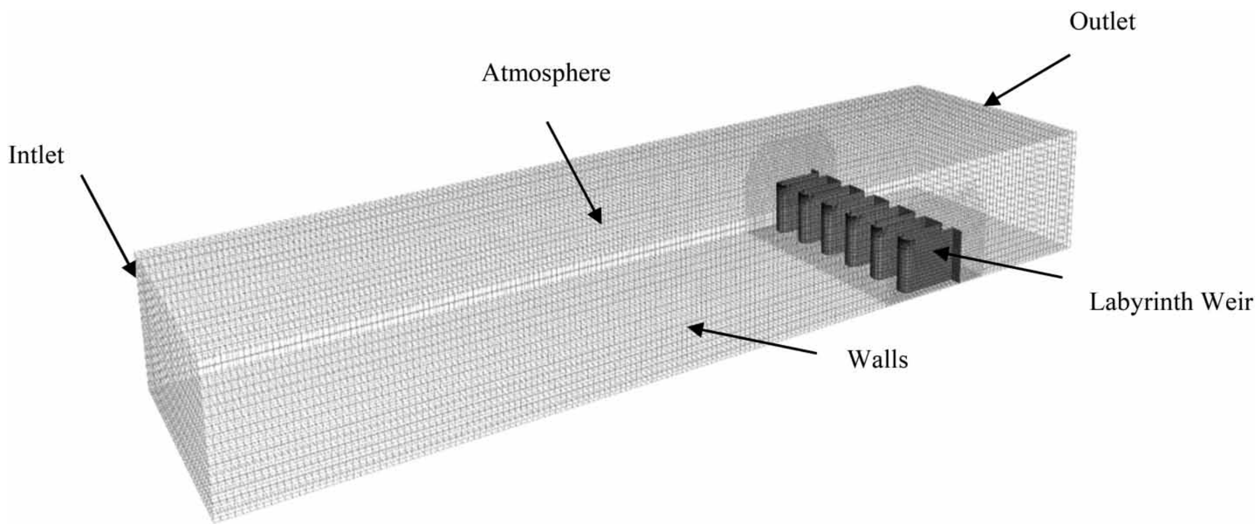

Figure 5 | Numerical model mesh and boundary surface.

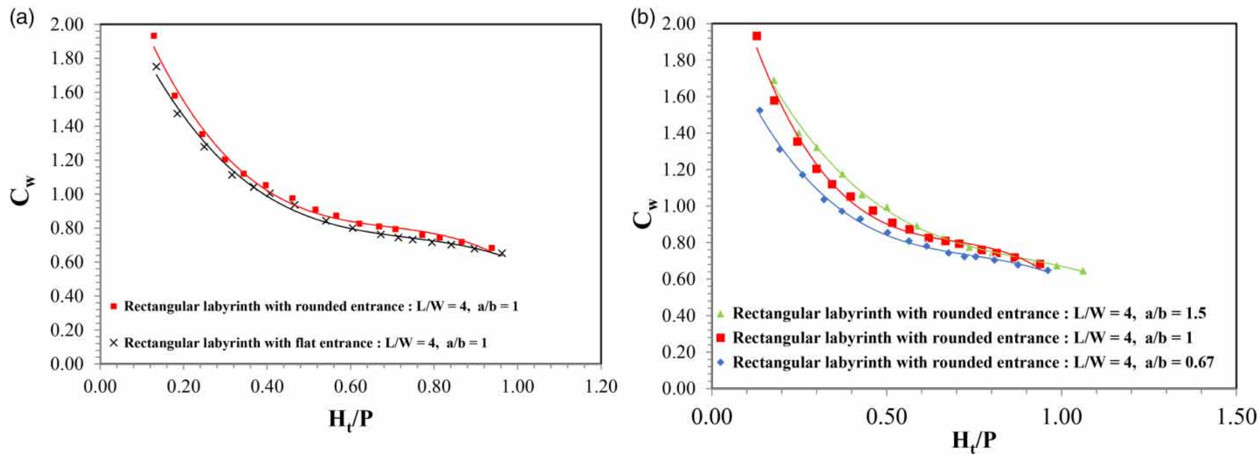

Figure 6 | Experimental results of: (a) entrance effect for a/b = 1, (b) alveoli width effect for a/b: 0.67,1 and 1.5.

The labyrinth with alveoli width equal to 0.67 had the lowest weir capacity compared to the other models. This is mainly due to the reduction in the inlet alveoli width. Thus, as a/b values increase then the inlet alveoli capacity increases also, thus allowing an increase in the labyrinth weir efficiency. According to the studies that used a rectangular labyrinth weir with a rounded entrance, the optimal value of the ratio a/b is close to 1.5. However, increasing a/b further led to the reduction of the outlet alveoli capacity and thus the global weir efficiency (Ben Said & Ouamane 2011, 2018).

3.2. Numerical results

3.2.1. Discharge coefficient and validation

The comparison of the discharge coefficient obtained from the CFD simulations and experimental tests are shown in Figure 7(a)–7(d) for ratio width a/b equal to 0.67,1,1.5 and 1 (for flat entrance), respectively. To quantify the difference between the computed and tested results, the estimation of the Mean Absolute Relative Error (MARE) was performed for the discharge coefficient. The MARE was computed using:

$$MARE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{O_i - S_i}{O_i}\right| \times 100 \quad (9)$$

where O_i and S_i are the experimental and simulated results, respectively, and N is the total number of data.

In general, a good agreement was found between the experimental and simulated results especially for models having the ratio a/b = 1, 1.5 and 1 (flat entrance). However, the less agreement obtained from the model has a/b = 0.67, mainly for low upstream heads, may be attributed to the geometric accuracy of the experimental model. The evaluated value of MARE for a/b = 0.67 was 6.31% (ranging from 0.06 to 18%), a/b = 1 was 5.56% (ranging from 0.7 to 9.6%), a/b = 1.5 was 6.32% (ranging from 3.16 to 9.68) and for the model having flat entrance and ratio a/b = 1 the MARE was 4.161% (ranging from 0.47 to 6.9%).

Figure 8 presents the experimental and numerical flow over the rounded rectangular labyrinth weir having a ratio a/b = 1.5 for rate of discharge Q = 80 l/s. It shows that the CFD model provides flow behaviors over the weir close to the ones obtained from the experimentation.

3.2.2. Entrance effect

In order to analyze the effect of the entrance on the labyrinth weir performance, discharge coefficients were numerically computed for two models with different inlet forms and having the same ratio a/b = 1.

As previously shown from the experimental results, the entrance shape has an effect on the discharge of the rectangular labyrinth weir, where the model designed with the rounded entrance is around 5% more efficient than the model with the flat entrance. The numerical results for the discharge coefficient are presented in Figure 9 for two configurations. This shows that the model designed with the rounded entrance is more efficient than the model designed with the flat entrance, which is in agreement with the results provided by the experimental model.

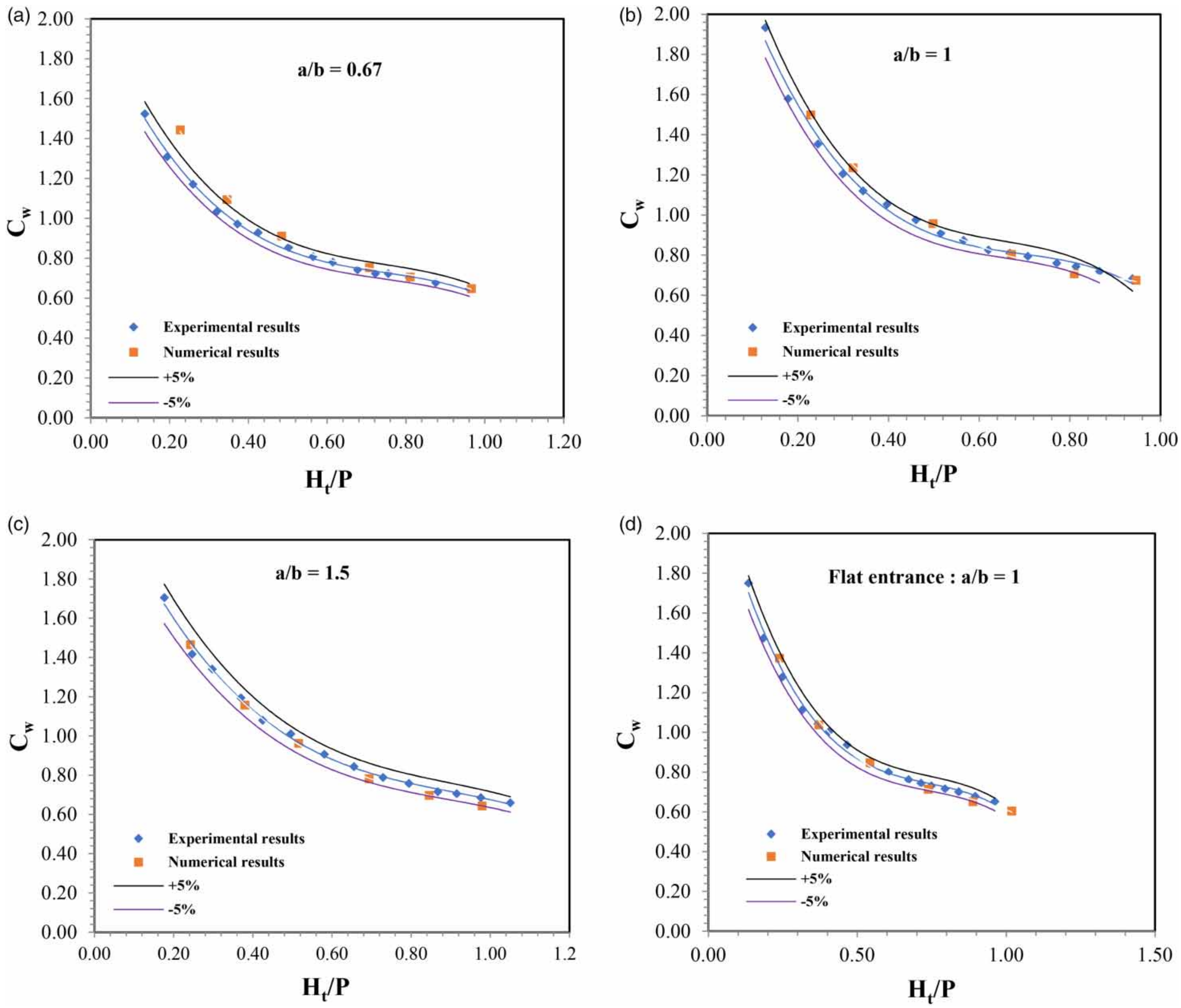

Figure 7 | Comparison between experimental and numerical results for: (a) a/b = 0.67, (b) a/b = 1, (c) a/b = 1.5 and (d) a/b = 1 (flat entrance).

To further analyze the effect of the entrance shape on the labyrinth weir capacity, simulated flow over two configurations was produced for $H_t/P = 0.3$. Figure 10 presents the flow streamlines for both flat and rounded entrances. It is clearly shown that the less flow contraction is provided by the model with the rounded entrance (Figure 10(b)) compared to the model with the flat entrance (Figure 10(a)). The use of the rounded entrance, providing a smoothly flow contraction, increases the rectangular labyrinth weir capacity by decreasing the energy losses at the inlet. Streamlines behavior at the inlet of the weir is presented in Figure 11(a) and 11(b) for flat and rounded shapes, respectively. The distribution of streamlines over the crest of two configurations is also presented in Figure 12. It was shown that the streamlines are distributed along the crest of the labyrinth designed with the rounded entrance (Figure 12(b)). However, the streamlines over the labyrinth with the flat entrance are mainly concentrated at the end portion of the crest as shown in Figure 12(a).

3.2.3. Alveoli width effect

Figure 13 compares the numerically computed coefficient of discharge for three rectangular labyrinth weirs with varying ratios (a/b = 0.67, 1 and 1.5). It was observed that the numerical model enables prediction of the influence of the alveoli width variations and shows that the model with the a/b ratio = 1.5 is more efficient compared with the model with 0.65 and provides a slightly larger capacity than the model with ratio a/b = 1. Globally, this observation is in agreement with the observation

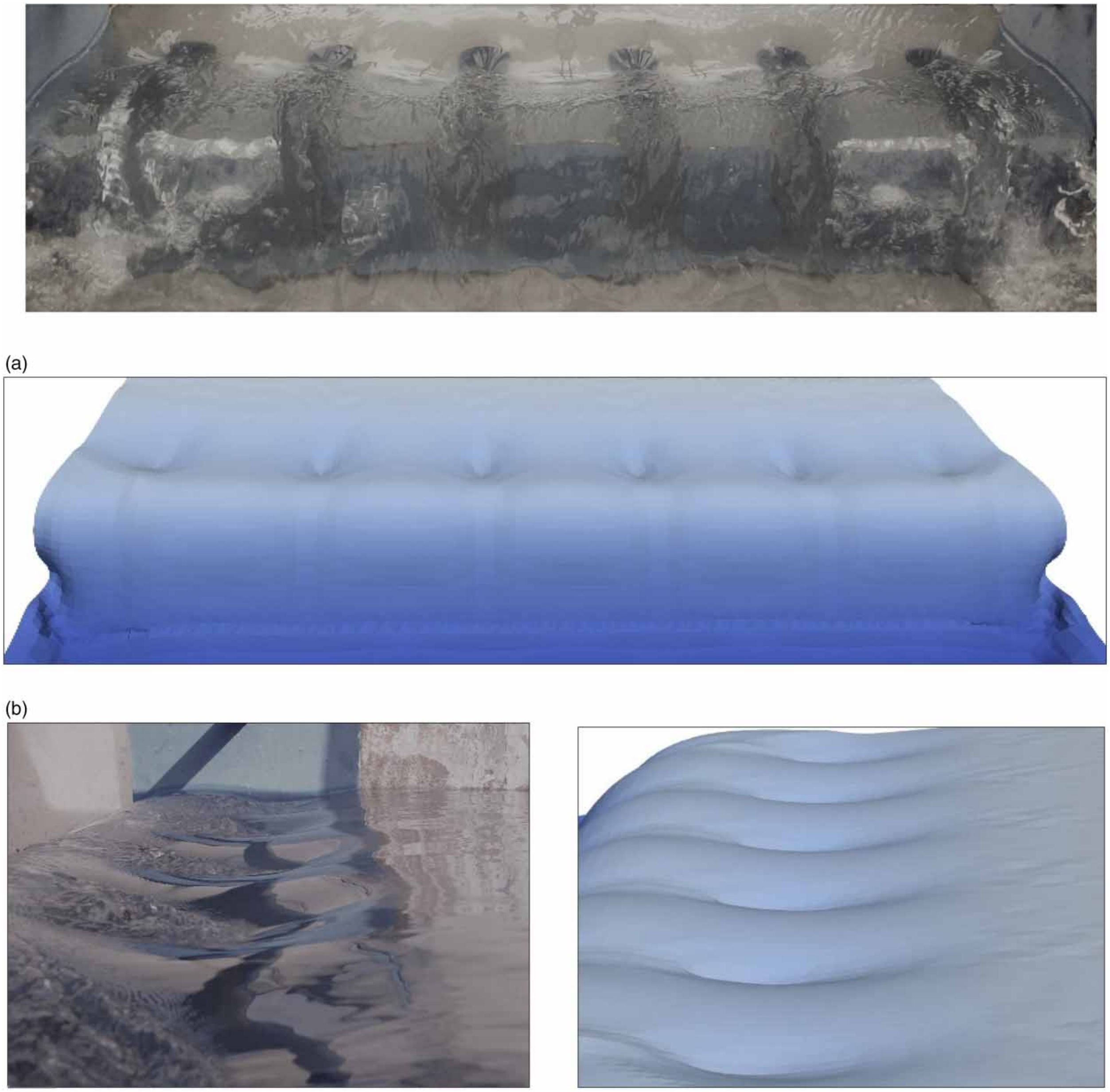

Figure 8 | Example of experimental and numerical flow over labyrinth weir for Q = 80 l/s: (a) view of the downstream, (b) view of upstream.

provided by the experimental model (Figure 6(b)). Streamlines over the rectangular labyrinth weir of three models have been numerically generated for the same upstream head $H_t/P = 0.3$ (Figure 14). A high concentration of streamlines can be clearly seen within the inlet alveoli of three simulated models, indicating that the majority of flow passes through the inlet alveoli.

A 3D flow also presented for three models in Figure 15, this highlights that the inlet alveoli of the model with a/b = 0.67 is poorly supplied by the flow compared to the models with ratios a/b = 1 and 1.5. These observations confirmed that the use of inlet width larger than the outlet width leads to an increase in the discharge capacity.

4. DISCUSSION

As previously discussed, the efficiency of the rectangular labyrinth weir designed with rounded entrance is larger than that obtained from the rectangular labyrinth weir having a flat entrance. The rounded shape enables an increase in the efficiency by 5% compared to the flat entrance. This can be attributed to the diminution in loss of energy at the entrance weir. Similar

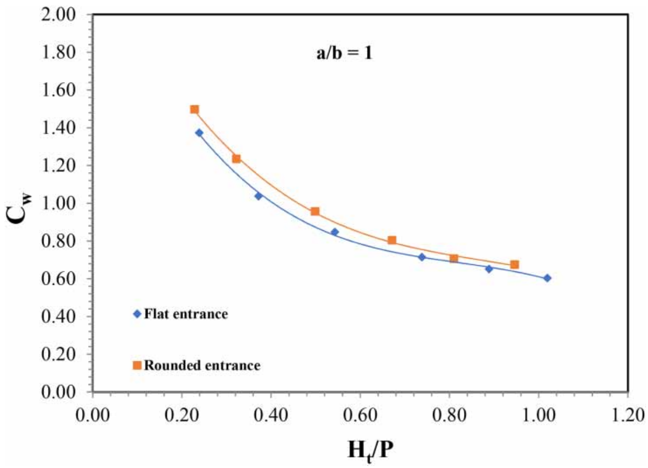

Figure 9 | Discharge coefficient obtained from numerical simulation for the entrance effect.

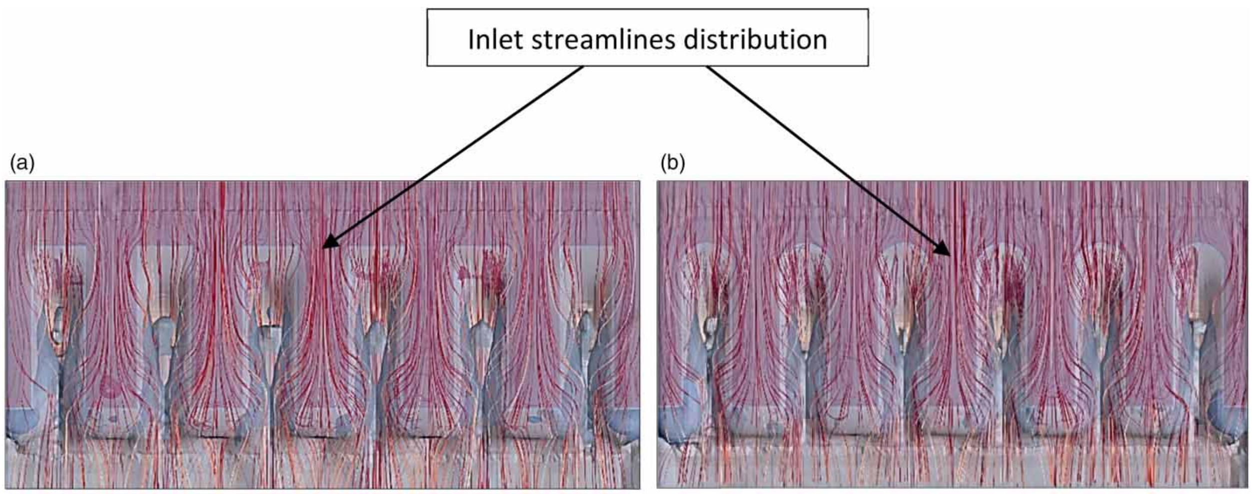

Figure 10 | 3D flow and streamlines over labyrinth weir, (a) with flat entrance and (b) with rounded entrance.

results were found by the experimental work carried out in reservoir application by Ouamane (2014) on two trapezoidal labyrinth weirs, one with a flat entrance and the second with a rounded entrance.

They confirmed that the trapezoidal labyrinth weir with rounded entrance provides higher discharge compared to the flat entrance. Their results indicate that the rounded shape provides a gain in efficiency of around 10%. Two reasons for this case were reported. First, adopting a rounded shape at the entrance facilitates the entrance of the flow in the inlet key, and thus the weir efficiency. Second, the rounded shape of the entrance enables elimination of the discontinuity points over the crest and thus to increase the effective crest length. Ouamane & Lempérière (2006) stated that installing a rounded or triangular nose under the upstream apex overhangs the Piano Key Weir, a new type of labyrinth weir, which increases the discharge capacity by 7%. This technic was used for designing Goulours and Saint-Marc dams (Laugier 2007, 2009). Anderson & Tullis (2013) also conducted tests to study the effect of the noses on the discharge capacity of Piano Key Weir. They indicated that noses with round shapes installed beneath the Piano Key Weir upstream apex overhangs increased the discharge capacity by up to 2.8%. They explain that the increase in efficiency by a decrease in inlet energy loss associated with the improved flow conditions at the entrance of the inlet.

The rectangular labyrinth weir designed with a larger inlet alveoli width produced the highest discharge capacity, as shown in the results of this study. The majority of the reported studies have been conducted to evaluate the effect of the alveoli width variation on the Piano Key Weir (Ouamane & Lempérière (2006), Machiels *et al.* (2011) and Andersonm & Tullis (2013).

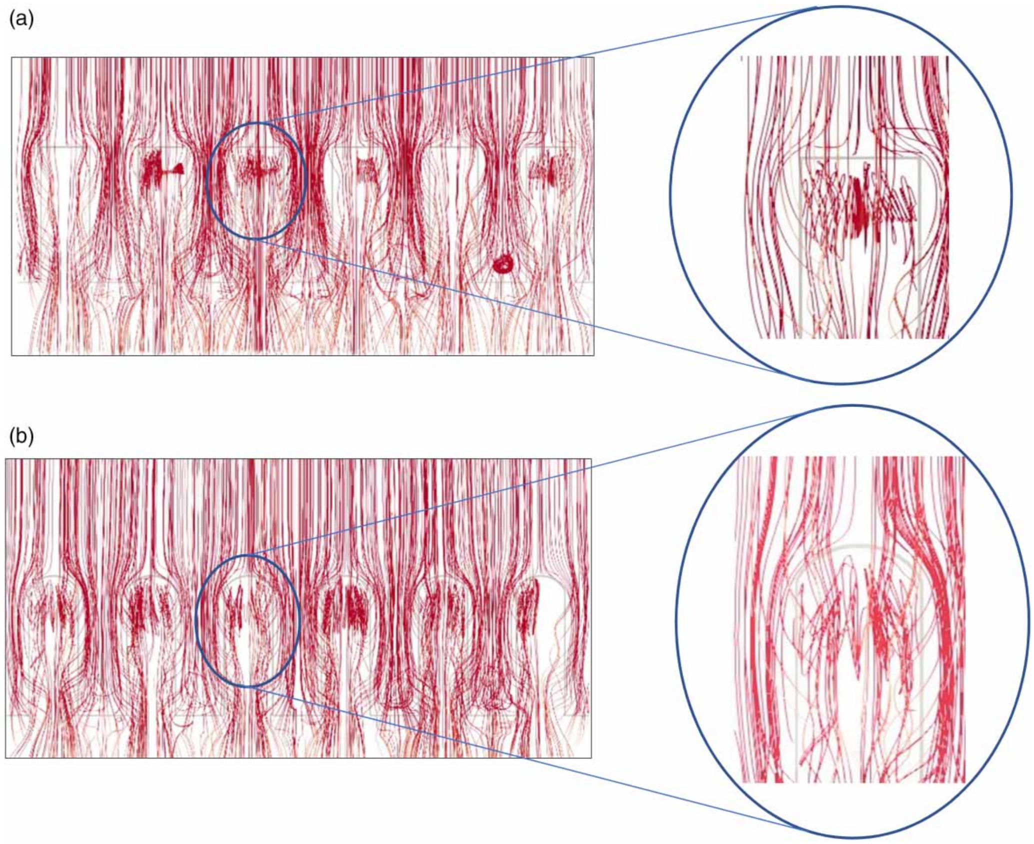

Figure 11 | Plan flow of streamlines over labyrinth weir, (a) with flat entrance and (b) with rounded entrance.

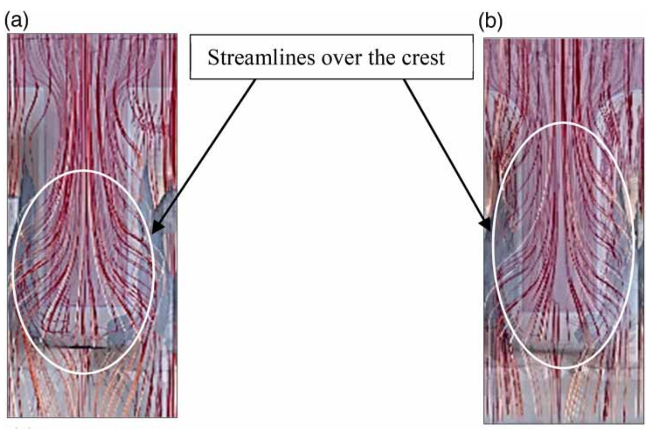

Figure 12 | Streamlines distribution over the crest of labyrinth weir (a) flat entrance, (b) rounded entrance.

However, to the author's knowledge, the studies by Ouamane & Ben Said (2010) and Kabiri-Samani *et al.* (2013) are the only two studies carried out to investigate the influence of the alveoli width on the capacity of the rectangular labyrinth weir. These two studies were performed in two different cases of approach flow conditions. In the first study, the labyrinth weir designed with a rounded entrance was installed in the reservoir, and in the second study, the model with the flat entrance was tested in channel conditions. Both studies reported that the discharge capacity increased with increasing the alveoli width ratio a/b.

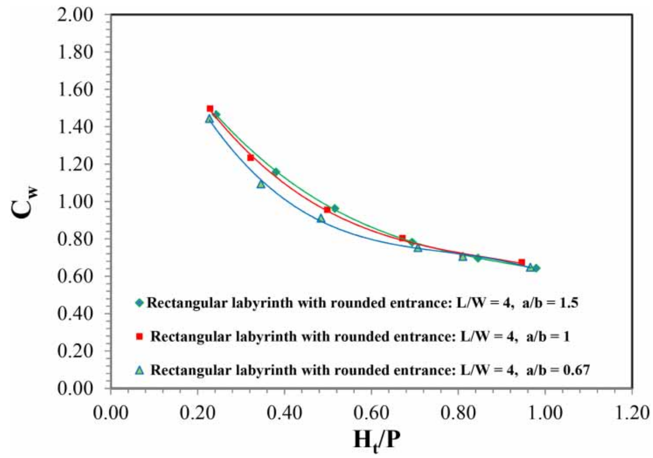

Figure 13 | Discharge coefficient obtained from numerical simulation for the alveoli width.

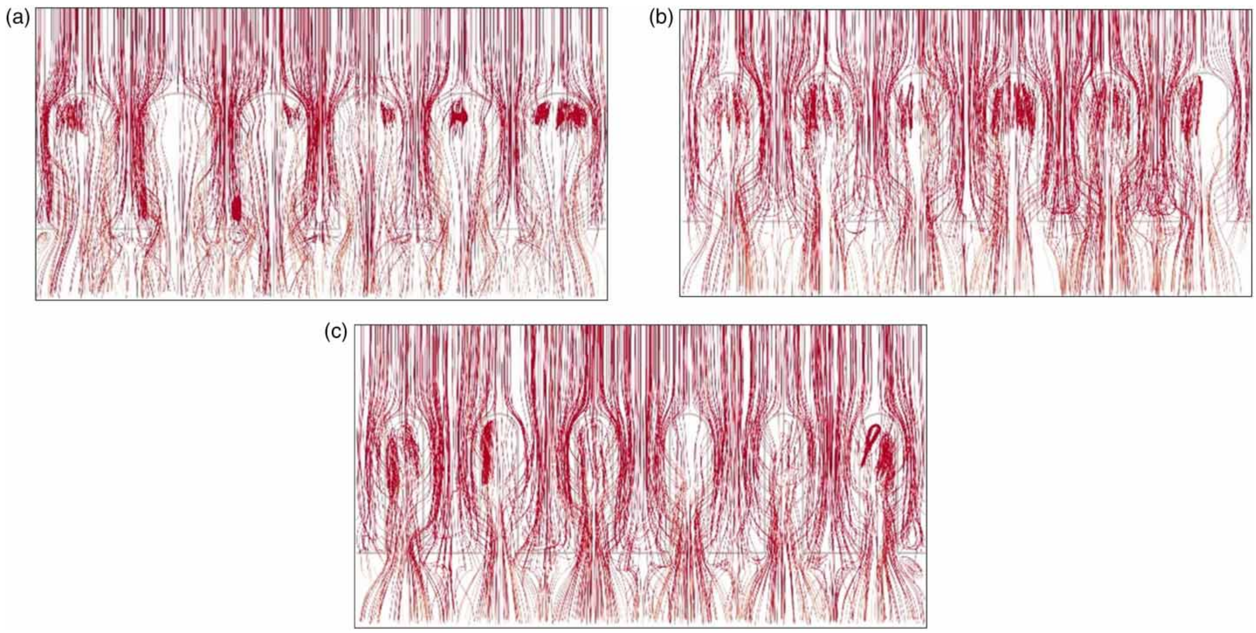

Figure 14 | Streamlines over the three tested models (a) $a/b = 0.67$, (b) $a/b = 1$ and (c) $a/b = 1.5$.

For low upstream heads H_t/P, the rectangular labyrinth weir leads to an increase in the efficiency by up to 10%. However, it is limited to 2% for higher upstream heads (Ouamane & Ben Said 2010). Kabiri-Samani *et al.* (2013) stated that a value of the ratio of the upstream apex length to the total spillway width (b/B) equal or greater than 0.1, for $H_t/P \leq 0.6$, was a sufficient value to limit the effect of the lateral nappe interference on the labyrinth efficiency. The highest efficiency in the current study corresponded to the ratio a/b equal to 1.5, which is within the optimal range, $a/b = 1.25$–1.5, as recommended by Andersonm & Tullis (2013) for designing the Piano Key Weir. The gain in efficiency of the model with $a/b = 1.5$ is related to the increase in the inlet alveoli capacity due to the increase in the inlet cross-section, which allows the entrance of the greatest flow in the inlet alveoli and thus increases the weir efficiency.

Machiels *et al.* (2014) attest that increasing the inlet cross-section caused a decrease in the flow velocity over the lateral crest, and thus it increased the contribution of the lateral crest in the overall efficiency of the Piano Key Weir.

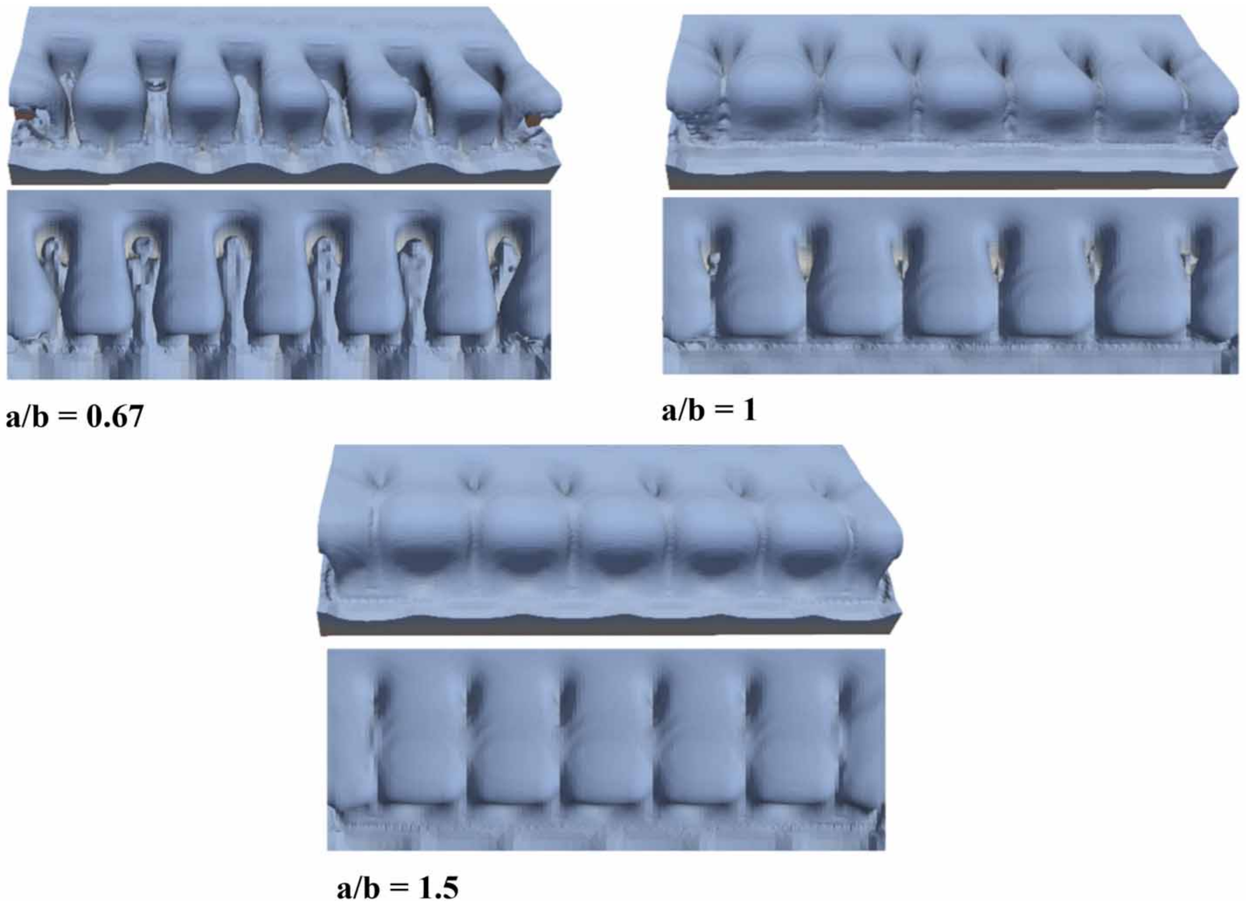

Figure 15 | Front and plan view of simulated flow over rectangular labyrinth weir with a ratio a/b equal 0.67, 1, and 1.5.

5. CONCLUSIONS

The purpose of this study was to investigate the influence of the geometric parameters on the rectangular labyrinth weir capacity. This study was conducted using an experimental and numerical model to investigate the entrance and the inlet and outlet width ratio a/b effect on the rectangular labyrinth weir efficiency. All models of the rectangular labyrinth weir have been tested in channel flow conditions for the same developed crest length ratio $L/W = 4$. The main conclusions of this study may be drawn as follows:

- The weir entrance shape significantly affects the discharge efficiency. A rectangular labyrinth weir designed with a rounded entrance was found to provide the highest discharge capacity. The gain in discharge efficiency is close to 5% compared to the labyrinth designed with flat entrance shape, this is for the whole range of upstream heads considered in this study ($H_t/P \leq 1$).
- For the rectangular labyrinth weir with a ratio $L/W = 4$ and designed with a rounded entrance, the results indicate that the increase of the ratio between the inlet and outlet alveoli width enables an increase in the discharge capacity of the labyrinth. The ratio $a/b = 1.5$ led to a 18% increase in the efficiency compared to the model with $a/b = 0.67$ and it is around 8% relative to the model with $a/b = 1$, mainly for upstream heads ($H_t/P \leq 0.5$).
- Using an inlet width larger than the outlet width enables an increase in the efficiency of the rounded rectangular labyrinth weir at no additional cost.
- Generally, a good agreement was found between the physical and numerical results. The obtained results indicated that the numerical model is able to compute the discharge capacity of the tested models of the rectangular labyrinth weir with mean average relative error of 4 and 6% for $a/b = 1$ (flat entrance) and $a/b = 0.67$, respectively.

- The numerical simulation using OpenFOAM CFD helps to further analyze the rectangular labyrinth weir efficiency by providing important information about the flow behavior, such as free surface profile and streamlines.
- The CFD simulations can be a useful tool to analyze and pre-design the labyrinth weir and help to reduce the cost of future weir designs.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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