Uniform flow and energy dissipation of hydraulic-jump-stepped spillways

Yu Zhou, Jianhua Wu, Fei Ma and Jianyong Hu

ABSTRACT

In skimming flow, a uniform flow can be achieved and the flow depth, velocity and air concentration remain constant if a stepped spillway is sufficiently long. In this study, physical model experiments were performed to investigate the uniform characteristics and energy dissipation of a hydraulic-jump-stepped spillway, which is a new type of stepped spillway for increasing the unit discharge capacity and energy dissipation. Based on the redefinition of uniform flow, experimental results show that at a given stepped spillway slope, a smaller height for the beginning of the uniform flow region, a greater uniform aerated flow depth and a greater uniform equivalent clear water flow depth can be obtained as compared with the traditional stepped spillway due to strong aeration in the aeration basin. Under the condition of uniform flow, the energy dissipation rate of stepped spillways can be estimated by the equivalent clear water flow depth with given inflow conditions. Compared with the traditional stepped spillway, the uniform flow over the hydraulic-jump-stepped spillway has a smaller specific energy, revealing that the hydraulic-jump-stepped spillway is more advantageous for dissipating energy, especially at large unit discharges.

Key words | aeration basin, energy dissipation, hydraulic-jump-stepped spillway, uniform characteristics

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INTRODUCTION

The stepped spillway has been widely used in hydraulic engineering as a simple structure for dissipating the energy of a flow (Sorensen 1985; Felder & Chanson 2011; Tabari & Tavakoli 2016). For a given stepped spillway, the flow pattern may be either nappe flow at small discharges, transition flow at intermediate discharges or skimming flow at large discharges (Chanson 1996, 2001; Zare & Doering 2012). In skimming flow, water skims as a coherent stream over the pseudo-bottom formed by the step edges, and a uniform flow can be attained if the stepped spillway is sufficiently long. A definition sketch of a skimming flow over a stepped spillway is shown in Figure 1, where q is the unit discharge, a and b are the length and height of each step, H_u is the stepped spillway height for the beginning of the uniform flow region, $h_{m,u}$ is the uniform aerated flow depth, and $h_{w,u}$ is the uniform equivalent clear water flow depth, respectively (Chanson *et al.* 2002). For the uniform flow region, hydraulic parameters such as the flow depth, velocity, and specific energy remain constant (Rajaratnam 1990; Chanson 1994b), which directly determine the residual energy at the end of the stepped spillway and consequently the total energy dissipation.

In recent years, numerous research contributions have been made on the uniform flow and energy dissipation of the stepped spillway. As regards the uniform flow, previous studies have mainly focused on the relationships between hydraulic and geometric parameters of stepped spillway and uniform flow characteristics, such as the beginning of

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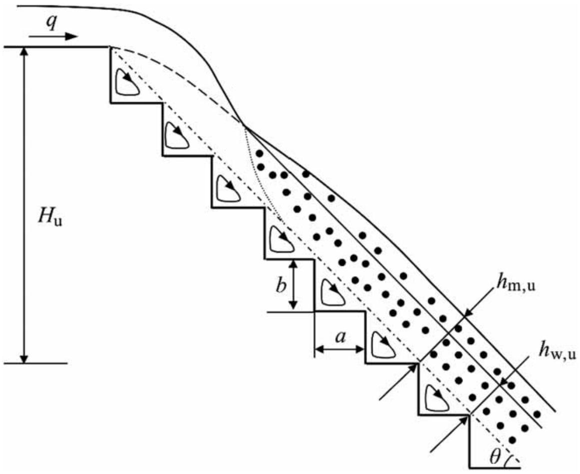

Figure 1 | Definition sketch of skimming flow over a stepped spillway.

the uniform flow region, uniform aerated flow depth and uniform equivalent clear water flow depth (Christodoulou 1999; Ohtsu *et al.* 2004; Pfister & Hager 2011). For energy dissipation, its mechanism is quite different due to different flow regimes over the stepped spillway (Chanson 1994a). The inflow conditions and geometric parameters of the stepped spillway, such as unit discharge (Sorensen 1985), upstream weir type (Silvestri *et al.* 2013), step size (Stephenson 1991) and the number of steps (Christodoulou 1993), affect the flow regimes of the stepped spillway, and then affect its energy dissipation. Because the skimming flow is predominant, as most prototype spillways operate at large discharges (Chanson 2001; Zare & Doering 2012), a focus on the energy dissipation of skimming flow is essential, especially for the condition of a uniform flow if the spillway is sufficiently long. For instance, with achieving a uniform flow, Chanson (1993) suggested that the energy dissipation rate can be estimated by friction coefficient. Peng *et al.* (2009) suggested aerated flow depth.

In order to increase the maximum safe unit discharge capacity and energy dissipation of stepped spillways, various new kinds of stepped spillway have been proposed (Pfister *et al.* 2006; Zamora *et al.* 2008; Qian *et al.* 2016). This research is based on a new kind of hydraulic-jump-stepped spillway, whose aeration basin is configured to provide pre-aerated flow for the stepped spillway by means of hydraulic jump, as shown in Figure 2 (Zhou & Wu 2017; Wu *et al.* 2018). The objective of this study is to experimentally investigate the uniform flow characteristics and energy dissipation of the hydraulic-jump-stepped spillway as compared with the traditional stepped spillway, which illustrates the advantages of the hydraulic-jump-stepped spillway in dissipating the energy of the flow.

EXPERIMENTAL SETUP AND METHODOLOGY

Experimental setup

All experiments were performed in the High-Speed Flow Laboratory of Hohai University, China. The experimental setup consisted of a pump, an approach conduit, a feeding basin, a hydraulic-jump-stepped spillway model of width $B =$ 27.50 cm and a flow return system (Figure 3). The hydraulic-jump-stepped spillway model included a 100.00 cm inflow channel, a weir, an aeration basin, a stepped spillway and a 10.00 m outflow channel, as shown in Figure 2. The weir

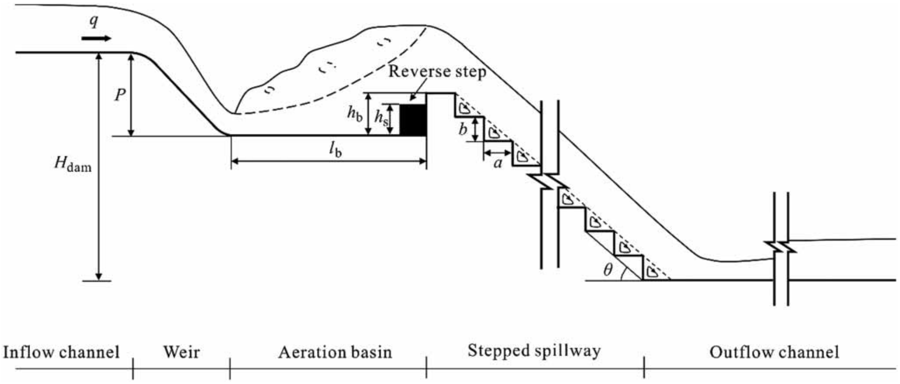

Figure 2 | Definition sketch of the hydraulic-jump-stepped spillway.

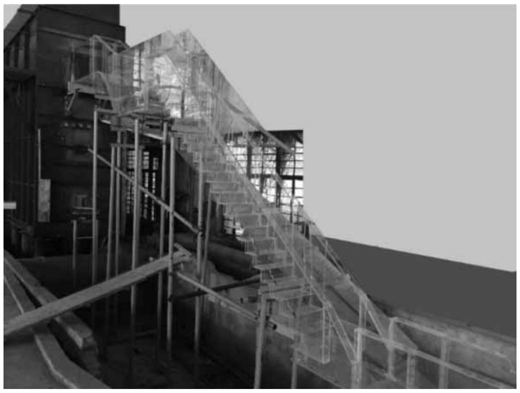

Figure 3 | Experimental setup.

Table 1 | Cases and geometric parameters of the aeration basin

Cases	l_b (cm)	h_b (cm)	Symbol
M11	87.5	18.0	■
M12	87.5	27.0	●
M21	105.0	18.0	◆
M22	105.0	27.0	▲
M31	122.5	18.0	□
M32	122.5	27.0	○

was the standard WES weir with the profile of $y = 1.1255x^{1.85}$; the height from the crest of the weir to the bottom of the aeration basin P was 63.00 cm; the slope of the overflow surface was 1V: 0.75H; and the toe of the weir was connected to the bottom of the aeration basin by an ogee curve with a radius of 27.50 cm. The aeration basin upstream of the stepped spillway was added to supply the aerated flow to the stepped spillway. A reverse step was placed in the aeration basin to stabilize the flow in the basin. The reverse step height $h_s = h_b - 0.5b$ with a length and width the same as that of the downstream single step. The stepped spillway was made up of 24 steps, where the step length a was 11.00 cm, the step height b was 9.00 cm, and the slope θ was 39.3°. The outflow channel was installed at the end of the stepped spillway to measure the equivalent clear water flow depth at the last step. The height from the crest of the weir to the bottom of the outflow channel H_{dam} was 2.61 m.

Experimental methodology

Table 1 shows the cases and the geometric parameters of the aeration basin, where l_b and h_b are the length and sill height of the aeration basin. A dimensionless parameter h_c/b is defined in this study to characterize the inflow condition, where $h_c = \sqrt[3]{(q^2/g)}$ is the critical depth, $q = Q/B$ is the unit discharge, and g is the acceleration of gravity. The inflow discharge Q varies from 0.023 m^3/s to 0.110 m^3/s (accordingly $h_c/b = 0.99–2.82$) and was measured by a V-notch weir with an accuracy of 2%.

The air concentration of the flow along the stepped spillway was measured by CQ6-2005 aeration apparatus, which has a sampling rate of 1,020 Hz and a period of 10 s, and the error is ±0.3%. Each air concentration probe was placed 2.5 cm apart from the step edge on the sidewall from the fifth step to the last step. Considering the aeration of the flow, the aerated flow depth was referred to as air concentration of 90%. By flow observation and measurement of air concentration profiles along the stepped spillway, the aerated flow depth was measured by a scale of 1 mm accuracy.

An outflow channel was installed in order to measure the uniform equivalent clear water flow depth $h_{w,u}$. It is known that once a uniform flow is attained in the stepped spillway, the flow depth will remain unchanged. In this case, the uniform equivalent clear water flow depth $h_{w,u}$ equals the equivalent clear water flow depth h_{w1} at the last step of the stepped spillway. A sketch of the measurement method of h_{w1} is given in Figure 4. Section 1-1 is located at the edge of the last step, and section 2-2 is selected about three to four times the step length away from the last step of the outflow channel, where the air is expected to be removed completely from the flow (Chinnarasri & Wongwises 2006). In this situation, the flow at section 2-2 can be assumed to be clear water. The clear water flow

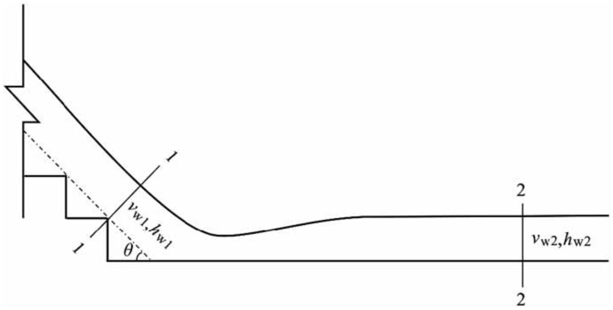

Figure 4 | Sketch of the measurement method of h_{w1}.

depth h_{w2} at section 2-2 was measured by point gauge with 0.1 mm accuracy, and the average flow velocity $v_{w2} = q/h_{w2}$ was obtained based on the unit discharge q.

Neglecting the energy losses between section 1-1 and 2-2, and considering the energy equation and continuity equation, we have:

$$h_{w1}\cos\theta + \frac{v_{w1}^2}{2g} = h_{w2} + \frac{v_{w2}^2}{2g} \quad (1)$$

$$v_{w1}h_{w1} = v_{w2}h_{w2} \quad (2)$$

The combination of Equations (1) and (2) will yield the equivalent clear water flow depth h_{w1} at section 1-1.

RESULTS AND DISCUSSION

Redefinition of uniform flow

Previously, two important conditions had to be satisfied in the formation of uniform flow. Firstly, a skimming flow occurs over the stepped spillway. Secondly, the flow depth, velocity and air concentration over the pseudo-bottom remain constant along the stepped spillway. Ohtsu *et al.* (2004) proposed that for a stepped spillway with a mild slope, the water-surface profile in the skimming flow is not always parallel to the pseudo-bottom but shows a wavy pattern. That is, the flow depths show significant changes even on each step, as shown in Figure 5. Bung (2011) showed that even though the water-surface profile is parallel to the pseudo-bottom, the presence of eddies in step niches leads to differences in the vertical distribution of the flow velocity and air concentration of each step. Based on these considerations, in this study, the uniform flow in the skimming flow is redefined as a flow over the stepped spillway where the flow depth, velocity and air concentration at the corresponding sections of each step remain constant.

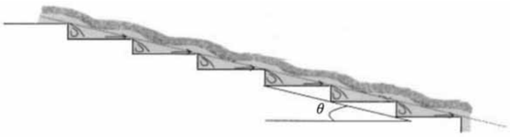

Figure 5 | Skimming flow over a stepped spillway with a mild slope (Ohtsu *et al.* 2004).

Uniform flow regime

Figure 6 shows photos of skimming flow over the hydraulic-jump-stepped spillway under different inflow conditions h_c/b for experiment M21. At a low h_c/b (e.g., $h_c/b = 0.99$, Figure 6(a)), the surface roller is made up of several vortices in the aeration basin, where the flow depth increases in the flow direction. In this situation, the aeration basin provides an outflow with only a small quantity of air for the stepped spillway. It is also noted that the flow depth decreases in the first several steps of the stepped spillway. Then, self-aeration occurs and entrained air develops in the flow. A uniform flow is attained as the flow is fully aerated, and thus the values of the hydraulic parameters at corresponding sections of each step are the same. However, at a high h_c/b (e.g., $h_c/b = 2.13$, Figure 6(b)), a strong hydraulic jump occurs in the aeration basin, resulting in a strong surface roller and, consequently, highly aerated flow to the stepped spillway. Similarly, the flow depth along the stepped spillway first decreases and then increases until a steady state is achieved. Thus, the uniform flow is attained under the

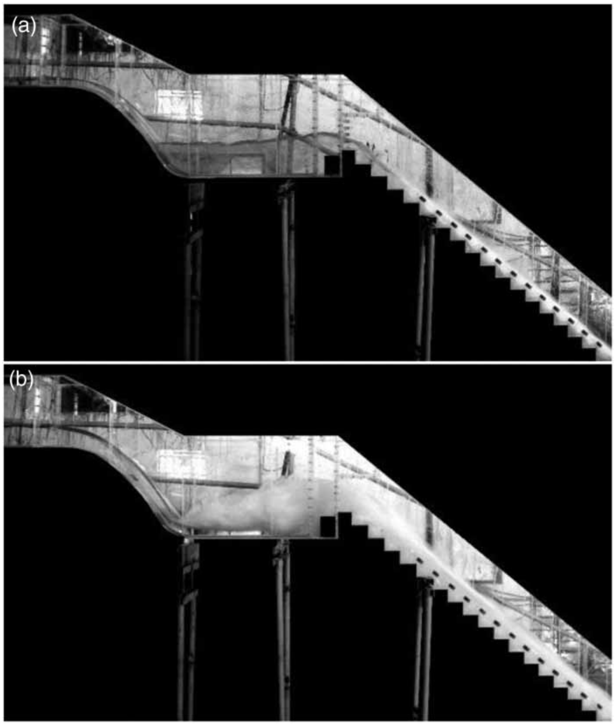

Figure 6 | Photographs of the skimming flow over the hydraulic-jump-stepped spillway (M21).

combined effects of the strong aeration in the aeration basin and the self-aeration in the stepped spillway.

Uniform flow characteristics

Figure 7 illustrates the uniform flow characteristics over the hydraulic-jump-stepped spillway as compared with the traditional stepped spillway (dashed line) of the same slope obtained in the study by Boes & Hager (2003a). The relationship between H_u/b and h_c/b is shown in Figure 7(a). It is found that for either the hydraulic-jump-stepped spillway or traditional stepped spillway, there is a good linear relationship between H_u/b and h_c/b, indicating that H_u/b increases linearly as h_c/b increases. Thus, as the unit discharge increases, the beginning of the uniform flow region is closer to the downstream end of the stepped spillway. H_u/b for the hydraulic-jump-stepped spillway can be described as follows according to the data in Figure 7(a) ($R^2 = 0.75$):

$$\frac{H_u}{b} = 3.13\frac{h_c}{b} + 12.43 \tag{3}$$

where $0.99 \le h_c/b \le 2.82$, $9.70 \le l_b/b \le 13.60$ and $2.00 \le h_b/b \le 3.00$.

It is also noted that for the traditional stepped spillway or the hydraulic-jump-stepped spillway, $H_u/b = 16$–18 when $h_c/b = 1.0$, indicating that at a small unit discharge, uniform flow is achieved at approximately the same location for the two stepped spillways, which can be attributed to the small effect of the hydraulic jump in the aeration basin. However, the difference between the two curves is larger as the unit discharge increases in Figure 7(a), which results in a smaller height for the beginning of the uniform flow region of the hydraulic-jump-stepped spillway compared with the traditional stepped spillway. The reason is as follows.

For the traditional stepped spillway, as the unit discharge increases, the flow depth along the spillway increases and the inception point of air entrainment moves downstream. After the occurrence of self-aeration, a large height is needed for the flow to become fully aerated, and the larger the unit discharge, the larger the height for the beginning of the uniform flow region. For the hydraulic-jump-stepped spillway, significant aeration has already taken place in the aeration basin and thus highly aerated flow is formed even at the first step, especially in the case of a large unit discharge. As a result, the height for the beginning of the uniform flow region is smaller in the hydraulic-jump-stepped spillway under the combined effects of the strong aeration in the aeration basin and the self-aeration in the stepped spillway.

Figure 7(b) shows the relationship between $h_{m,u}/b$ and h_c/b. It is noted that at a given stepped spillway slope, there is also a good linear relationship between $h_{m,u}/b$ and h_c/b for both the hydraulic-jump-stepped spillway and traditional stepped spillway, but the former curve has a larger slope than the latter. At around $h_c/b = 1.0$, $h_{m,u}/b = 0.70$–0.80, indicating that there is only a small difference in the uniform flow depth between the two spillways. However, $h_{m,u}/b$ of the hydraulic-jump-stepped spillway increases more rapidly as h_c/b increases due to the difference between the two curves. The $h_{m,u}/b$ for the hydraulic-jump-stepped spillway can be described as follows ($R^2 = 0.96$):

$$\frac{h_{m,u}}{b} = 0.82\frac{h_c}{b} \tag{4}$$

where $0.99 \le h_c/b \le 2.82$, $9.70 \le l_b/b \le 13.60$ and $2.00 \le h_b/b \le 3.00$.

Figure 7(c) illustrates the relationship between $h_{w,u}/b$ and h_c/b. Again, there is a good linear relationship between $h_{w,u}/b$ and h_c/b for both the hydraulic-jump-stepped spillway and traditional stepped spillway with a given stepped spillway slope, and the former increases more rapidly than the latter. For example, $h_{w,u}/b$ is 0.30 and 0.60 at around $h_c/b = 1.0$, and 0.80 and 1.70 at $h_{w,u}/b = 2.80$ for the traditional stepped spillway and hydraulic-jump-stepped spillway, respectively, indicating that the equivalent clear water flow depth of uniform flow over the hydraulic-jump-stepped spillway is higher than that over the stepped spillway especially for large unit discharges. The $h_{w,u}/b$ for the hydraulic-jump-stepped spillway can be described as follows ($R^2 = 0.96$):

$$\frac{h_{w,u}}{b} = 0.64\frac{h_c}{b} \tag{5}$$

where $0.99 \le h_c/b \le 2.82$, $9.70 \le l_b/b \le 13.60$ and $2.00 \le h_b/b \le 3.00$.

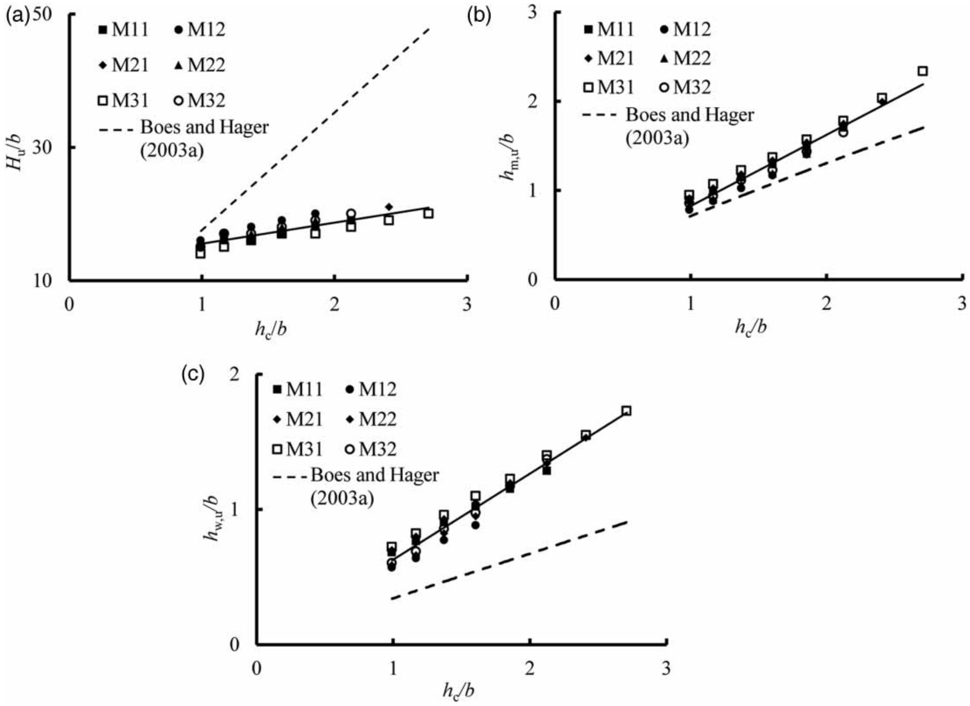

Figure 7 | Uniform flow characteristics over the hydraulic-jump-stepped spillway.

Energy dissipation of the uniform flow

In a skimming flow, the equivalent clear water flow depth remains unchanged once uniform flow conditions are attained. In this case, the specific energy E_s can be described as follows (Boes & Hager 2003b):

$$E_s = h_{w,u} \cos\theta + \frac{q^2}{2gh_{w,u}^2} \quad (6)$$

The specific energy E_s in Equation (6) is essentially the sum of the pressure head $h_{w,u}\cos\theta$ and the velocity head $q^2/(2gh_{w,u}^2)$ with respect to the pseudo-bottom. The flow from the beginning of the uniform flow region to the end of the hydraulic-jump-stepped spillway is a uniform flow, and its specific energy E_s remains unchanged. Under this condition, the specific energy E_s equals the residual energy E_{res} at the end of the stepped spillway (i.e. $E_{res} = E_s$). Here, the residual energy E_{res} is related only to the uniform flow characteristics and the slope of the stepped spillway rather than the height and length of the stepped spillway. Thus, the energy dissipation rate η for the stepped spillway can be calculated from Equation (7):

$$\eta = \frac{E_1 - E_{res}}{E_1} = \frac{E_1 - E_s}{E_1} \quad (7)$$

where E_1 and E_{res} are the total energy head and residual energy head above the bottom of the outflow channel. Under uniform flow conditions, the residual energy E_{res} at the end of the stepped spillway can be expressed as follows:

$$E_{res} = h_{w,u} \cos\theta + \frac{q^2}{2gh_{w,u}^2} \quad (8)$$

For a given stepped spillway with a slope θ, if the inflow conditions (e.g., the unit discharge q and the total energy head E_1) are known and the equivalent clear water flow depth $h_{w,u}$ of the uniform flow can be determined, the energy dissipation rate η in the skimming flow can be calculated from Equations (7) and (8).

Figure 8 shows the relationship between E_s/b and h_c/b for the uniform flow over the hydraulic-jump-stepped

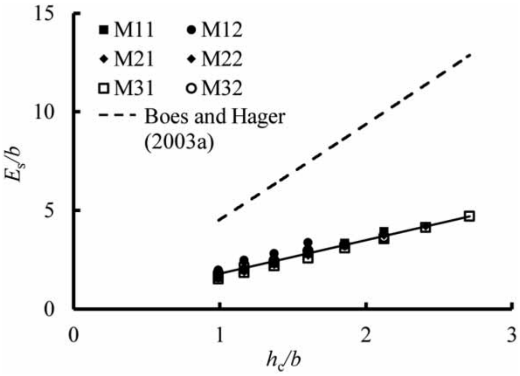

Figure 8 | Relationship between E_s/b and h_c/b.

spillway compared with the traditional stepped spillway (dashed line) of the same slope. It shows that at a given spillway slope, there is a linear relationship between E_s/b and h_c/b for both the hydraulic-jump-stepped spillway and traditional stepped spillway, and the former has a smaller E_s/b than the latter with increasing h_c/b. Thus, the residual energy difference between the two spillways increases as the unit discharge increases once uniform flow is attained, indicating that the hydraulic-jump-stepped spillway is more effective than the traditional stepped spillway in dissipating energy, especially for large unit discharges. The E_s/b for the hydraulic-jump-stepped spillway can be fitted to ($R^2 = 0.94$):

$$\frac{E_s}{b} = 1.7\frac{h_c}{b} + 0.08 \tag{9}$$

where $0.99 \leq h_c/b \leq 2.82$, $9.70 \leq l_b/b \leq 13.60$ and $2.00 \leq h_b/b \leq 3.00$.

CONCLUSIONS

(1) Uniform flow has been redefined accounting for the presence of eddies in step niches and a wavy pattern with a mild slope.
(2) At a given slope, the height for the beginning of the uniform flow region of the hydraulic-jump-stepped spillway compared with the traditional stepped spillway is smaller due to strong aeration in the aeration basin, and the uniform aerated flow depth and the uniform equivalent clear water flow depth are greater.
(3) If the inflow conditions are known, the energy dissipation rate can be estimated by the equivalent clear water flow depth once a uniform flow is attained. Compared with the traditional stepped spillway, the uniform flow over the hydraulic-jump-stepped spillway has a greater equivalent clear water flow depth and thus a smaller specific energy, resulting in a significant increase in energy dissipation, especially at a large unit discharge.

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