Discharge coefficients for ogee weirs including the effects of a sloping upstream face

Farzin Salmasi and John Abraham

ABSTRACT

Discharge coefficients (C_0) for ogee weirs are essential factors for predicting the discharge-head relationship. The present study investigates three influences on the C_0: effect of approach depth, weir upstream face slope, and the actual head, which may differ from the design head. This study uses experimental data with multiple non-linear regression techniques and Gene Expression Programming (GEP) models that are applied to introduce practical equations that can be used for design. Results show that the GEP method is superior to the regression analysis for predicting the discharge coefficient. Performance criteria for GEP are $R^2 = 0.995$, RMSE = 0.021 and MAE = 0.015. Design examples are presented that show that the proposed GEP equation correlates well with the data and eliminates linear interpolation using existing graphs.

Key words | discharge coefficient, GEP, ogee weir, regression analysis, upstream face

Farzin Salmasi (corresponding author)
Department of Water Engineering, Faculty of Agriculture,
University of Tabriz,
P.O. Box: 5166616471, Tabriz,
Iran
E-mail: *salmasi@tabrizu.ac.ir*

John Abraham
School of Engineering
University of St. Thomas, Minnesota,
2115 Summit Avenue, St. Paul, Minnesota 55105,
USA

SYMBOLS

The following symbols are used in this paper:

C_0 = discharge coefficient for free (uncontrolled) flow condition with vertical upstream face ($m^{0.5}/s$);

C_i = discharge coefficient for free flow condition, with sloping face in upstream ($m^{0.5}/s$);

g = acceleration due to gravity;

H = reprehensive horizontal distance in upstream weir slope;

H_0 = design head over the crest (m);

H_e = actual head (other than the design head) being considered on the crest (m);

h_0 = upstream water depth above the crest in design discharge (m);

I = angle of the upstream face in ogee weir with respect to vertical direction (degrees);

L = effective length of the crest (m);

Q = design discharge for ogee spillway (m^3/s);

P = ogee spillway height (m);

V = reprehensive vertical distance in upstream weir slope;

V_a = approach velocity in m/s $\left(V_a = \dfrac{Q}{L(P + H_0)}\right)$

doi: 10.2166/ws.2020.064

INTRODUCTION

An ogee spillway is one of the weir types that is used in many dam spillway types like diversion, earth, gravity, rock fill, buttress, and arch dams. It is the most common type and is typically constructed from concrete. Spillways can be located over a conventional gravity dam, an arch gravity dam, and a roller compacted concrete (RCC) dam. If the dam is an embankment, then the spillway can be located over the right or left abutments and is typical made from hard material like concrete.

Flow over an ogee spillway is dependent on the discharge coefficient (C_0). These spillways are designed based on a specific discharge, referred to as the design discharge (Q_{design}).

There are two kinds of ogee spillways: (i) – ogee spillway with gates on the crest that can control the discharge over the spillway and (ii) – ogee spillway without gates. These spillways do not have any control of discharge. The advantages of the un-gated or uncontrolled crest are the elimination of the flow control devices and the lower maintenance and repairs.

The shape of the ogee spillway depends upon the head, the inclination of the upstream face of the overflow section, and the height of the overflow section above the floor of the entrance channel (USBR 1987).

The discharge over an uncontrolled overflow ogee crest is given by Equation (1) (Kim & Park 2005):

$$Q = C_0 L H_0^{1.5} \tag{1}$$

where Q is the design discharge (m^3/s), C_0 is the variable discharge coefficient for free (uncontrolled) flow conditions (m$^{0.5}$/s), L represents the effective length of the crest (m), and H_0 symbolizes the design head or actual head being considered (m), including the velocity of approach head, h_a (m). In Equation (1), $C_0 = (2/3)C\sqrt{2g}$ where C_0 is the weir coefficient (dimensionless) and g is gravity acceleration (m/s^2).

The discharge coefficient, C_0, is influenced by a number of factors. These factors are: the effect of the approach depth, the effect of heads different from the design head, the effect of the upstream face slope, the effect of the downstream apron interference, and the effect of the downstream submergence. Influences of the first three factors are the subject of the present study.

USBR (1987) proposed discharge coefficient graphs for ogee crests with different geometries. These graphs (for gated, ungated and various upstream slope conditions) have been used by hydraulic designers for many years. The gap in knowledge is that there is no simple equation to combine these graphs and thus eliminate linear interpolation among several curves. The objective of present study is to propose an equation for estimating discharge coefficients. For this purpose, multiple regression techniques and Gene Expression Programming (GEP) models are applied with dimensionless parameters. The experimental data are from USBR (1987). The information for discharge coefficient includes 202 data points, obtained via experiments on several ogee spillways. The discharge coefficient is calculated using Equation (1), i.e. $C_0 = Q/(LH_0^{1.5})$.

Another reason for eliminating linear interpolation among several curves relates to Moody's (1947) diagram. Based on this diagram, friction factor (f) in pipes depend on two dimensionless parameters: Reynolds number (R_e) and the relative roughness (e/D). Pipe roughness is denoted with e and pipe diameter is refer with D. However, the mathematical formulation to find f, includes an empirical equation that is well-known as the Colebrook-White equation. This equation is such that the f factor appears on both sides of the equation (Salmasi *et al.* 2012) and requires iteration for its solution

Effect of approach depth

The approach velocity (V_a) for high sharp-crested weirs, is low and the lower nappe from these weirs leads to high contraction. Increasing discharge or reducing weir height causes an increase in V_a and this phenomenon reduces lower nappe contraction.

For sharp-crested weirs whose heights are more than one-fifth of the total head ($P/H_0 > 0.2$), the discharge coefficient is constant with a value of about 1.82. For weir heights less than about one-fifth the head, the contraction of the flow becomes increasingly suppressed and the crest coefficient decreases. When the weir height becomes zero, the contraction disappears and the overflow weir becomes a channel or a broad-crested-weir, for which the theoretical discharge coefficient is 1.70. Figure 1 shows variation of discharge coefficient for vertical-faced ogee crests with height-over-head ratio (P/H_0).

Effect of the heads on discharge coefficient

Higher discharge than the design discharge causes negative pressure over the spillway bottom. This is because the nappe tends to disconnect from the spillway bottom. Lower discharge than the design discharge causes positive pressure over the spillway bottom. This is because the nappe has a tendency to connect to the spillway bottom. Figure 2

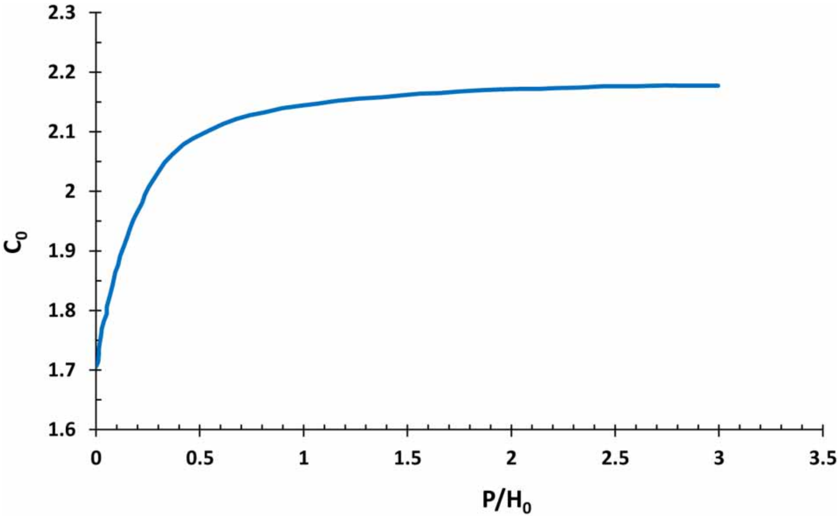

Figure 1 | Vertical-faced discharge coefficients for ogee crests (USBR 1987). *Note*: values of C_o is based on SI system units ($m^{0.5}/s$).

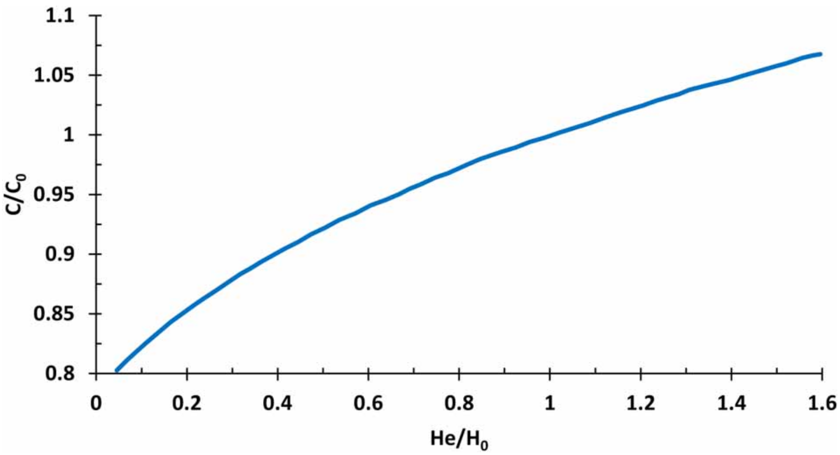

Figure 2 | Discharge coefficients for other than the design head (USBR 1987).

provides the changes of the dimensionless C/C_0 versus values of H_e/H_0. The symbol H_e is the actual head.

Influence of face slope

With small values of P/H_0, sloping the upstream face of the ogee spillway increases the discharge coefficient. The effect of upstream slope on discharge coefficient decreases for large ratios of P/H_0. Figure 3 compares the ogee spillway discharge coefficient with an inclined face (C_i) to that for a crest with a non-inclined face (C_v) as related to values of P/H_0.

Figures 4 and 5 show the application of ogee spillways in storage and diversion dams respectively.

Savage & Johnson (2001) carried out a study of flow characteristics over a standard ogee-crested spillway by means of physical and numerical models. Observation of pressure and discharge showed proper agreement between the physical and numerical models. Tullis (2011) evaluated the performance of submerged ogee coefficients as related to upstream and downstream weir height and flow discharge. Results showed that the USBR (1987) dimensionless relationships developed by Bradley (1945) underestimated the effect of submergence (S) on C_s for multiple ogee crest weirs.

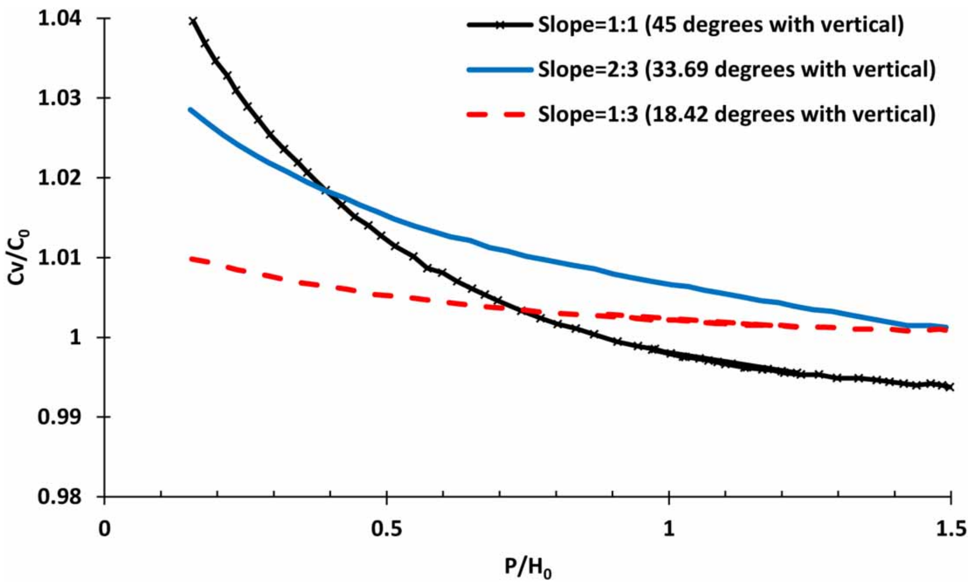

Figure 3 | Discharge coefficients for ogee spillways including a sloping upstream face (USBR 1987).

Figure 4 | An ogee spillway in Walayar reservoir, India.

Tullis (2011) reported that the weak relationship between observed and predicted C_s values might be caused by differences between h_d measurement positions.

Bradley (1945) measured h_d nearer to the weir than in the Tullis (2011) study. Tullis & Neilson (2008) showed that for submergence levels less than 0.70, head-discharge correlations were independent of the tail elevation. On the other hand, for higher submergence, the trends were reversed.

Madadi *et al.* (2014) investigated flow characteristics of a trapezoidal broad-crested weir. The results showed that a

Figure 5 | An ogee spillway in a diversion dam in Iran.

reduction of the upstream slope avoids the progress of a flow separation zone. A decrease in the upstream slope to 21° resulted in an increase in the discharge coefficient. The changes were as large as 10%. Furthermore, the separation relative length and height decreased up to 80% and 95%, respectively.

Al-Khatib & Gogus (2014) studied discharge in rectangular compound broad-crested weirs using multiple regression equations. In that study, the dependencies of the discharge coefficient and upstream velocity on different operating parameters were investigated based and reported using dimensionless ratios. Results showed when the head in the upstream section is given, the flow discharge can be evaluated with an error of less than ~ 5%.

Guven *et al.* (2013) studied flow over broad-crested weirs and through box culverts. The latter's performance was superior for conveying water.

Salmasi (2018) presented equations for estimating discharge coefficients in ogee weirs with consideration of downstream stilling basin elevation and submergence ratio. Salmasi (2018) refers to the ease of these equations compared to traditional charts.

Estimation of discharge coefficient is also important in other hydraulic structures like sluice gates and other weirs. Recent investigations for determining discharge coefficients in radial gates include the study of Salmasi *et al.* (2019), which refers to the effect of sills on discharge. In another study, Salmasi (2019) found the discharge coefficient for circular labyrinth weirs. Akbari *et al.* (2019) predicted discharge coefficients for gated piano key weirs using experimental and artificial intelligence (AI) methods.

Some aspects of the hydrodynamics of rectangular broad-crested porous weirs comprise determination of the discharge coefficient. Flow can occur both through and over these weirs and this phenomenon changes the discharge coefficient values (Mohamed 2010; Salmasi & Abraham 2020).

The goal of this study is to create an accurate method for predicting C_0 for ogee weirs. To introduce the head-discharge relationship, designers need to use Figures 1–3 and employ linear interpolation among the slopes in Figure 3. This approach results in estimations of C_0 and head-discharge relationship with some error. Here, the effects of three factors: P/H_0 ratios, heads that differ from the design head (H_e/H_0), and the effect of the upstream face slope (I) are investigated using regression analysis and GEP modeling. The goal is to provide a methodology that is independent of design charts provided by USBR (1987). To our best knowledge, there is no equation for predicting C_0 values in ogee weirs with consideration of the three above factors.

MATERIAL AND METHODS

Geometric and hydraulic variables

Figure 6 denotes hydraulic parameters in an ogee weir with heads greater/less than the design head. In the figure, the symbol *El C* represents crest elevation from an arbitrary datum, P is ogee weir height, H_0 is design head, $h_a = V_a^2/2g$ is approach velocity head, H_{e1} and H_{e2} are actual heads being considered on the crest, the approach velocity V_a is equal to $V_a = Q/L/(P + H_0)$, L is the weir length and Q is the design discharge. Figure 5 indicates geometric and hydraulic variables in an ogee weir with sloping upstream face. In Figure 7, I is the upstream face of the weir inclination respect to the vertical direction. In the following equations, I is in degrees. The other parameters have been defined previously.

Regression analysis

In this study, the effective parameters are: C_0 versus P/H_0 (effect of depth of approach), C/C_0 versus H_e/H_0 (head differing from the design head) and C_i/C_v versus P/H_0

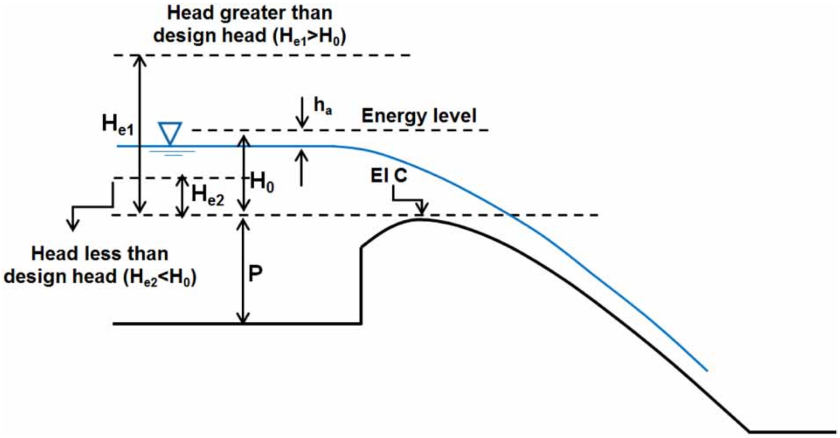

Figure 6 | Geometric and hydraulic variables in an ogee weir with head greater/less than design head (vertical sloping face).

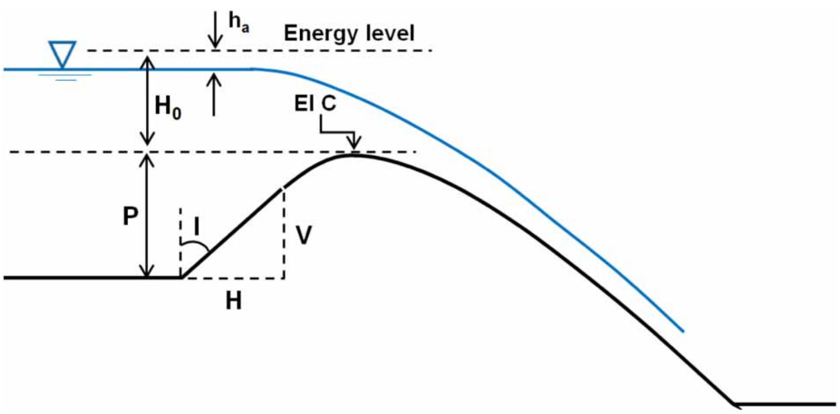

Figure 7 | Geometric and hydraulic variables in an ogee weir with sloping upstream face in design head.

(upstream sloping face effect). A functional relationship among parameters is described in Equation (2) (USBR 1987):

$$C_0 = f\left(\frac{P}{H_0}, \frac{H_e}{H_0}, I\right) \tag{2}$$

where I is the weir upstream face inclination with respect to the vertical direction (in degrees) and other parameters were defined previously.

All test data are from different geometries of ogee spillways. The number of data points used in this study is 202 (Appendix I) and the domains of variation for dimensionless parameters in Equation (2) are: $0 < P/H_0 < 3$, $0 < I < 45$ and $<0.05H_e/H_0 < 1.6$. Thus, different geometries include different values for P and I parameters and different hydraulic conditions relate to H_0 and H_e.

In the present study, a commercially available computer code (SPSS) software version 22 (SPSS 2013) and Curve-Expert Professional (2012) software are used for classical regression analysis.

Application of GEP

The software GeneXproTools (2011), version 4.0 (Ferreira 2001a, 2001b) was used in this study for prediction of the discharge coefficient.

The GEP considers crossover and mutation operators to be 'winners' (children) and then compete in natural selection. Crossover operations preserve features from one generation to the next. On the other hand, mutations lead to random changes in generations.

Performance criteria

In this study, to evaluate the accuracy of the proposed models (regression and GEP), the three indicators, (1) root mean square error (RMSE), (2) determination coefficient (R^2) and (3) mean absolute error (MAE), are used. They are presented in Equations (3)–(5) (Akbari *et al.* 2019):

I. Root mean square error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (y_p - y_0)^2}{N}} \tag{3}$$

II. Determination coefficient (R^2)

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y_0 - y_p)^2}{\sum_{i=1}^{N} (y_0 - \bar{y}_0)^2} \tag{4}$$

III. Mean absolute error (MAE)

$$MAE = \frac{\sum_{i=1}^{N} |y_0 - y_p|}{N} \tag{5}$$

In these equations, N represents the number of observations; y_0 symbolizes observed data; the term y_p represents predicted data; $\bar{y}_0$ is the value of the mean from the observations; and $\bar{y}_p$ is the mean of the predictions. As previously noted, the total number of data points are 202, of which 141 (70%) are used for training and 61 (30%) for testing. In Appendix I, these data points are presented.

RESULTS AND DISCUSSION

Regression equations

In this study, to derive equations with one dependent variable, CurveExpert Professional (2012) software is used and for equations with two or more dependent variables (multiple regression), SPSS (2013) software and GeneXproTools (2011) software are used. Variation of discharge coefficients (C_0) for vertical-faced ogee crest against P/H_0 (Figure 1) are presented in Equation (6) ($R^2 = 0.999$):

$$C_0 = 1.715 + \frac{0.478\left(\frac{P}{H_0}\right)^{1.258}}{0.113 + \left(\frac{P}{H_0}\right)^{1.258}} \tag{6}$$

Variation of relative discharge coefficients (C/C_0) versus a head other than the design head (Figure 2) is obtained from Equation (7) ($R^2 = 0.999$):

$$\frac{C}{C_0} = 0.255 + 0.745\left(\frac{H_e}{H_0}\right)^{0.176} \tag{7}$$

Variation in discharge for ogee-shaped crests with an upstream slope (Figure 3) are calculated using Equation (8):

$$\frac{C_i}{C_v} = 0.00022(I) + 0.998\left(\frac{P}{H_0}\right)^{-0.003} \tag{8}$$

Table 1 shows prediction errors for Equations (6)–(8). Equation (8) demonstrates that a non-linear regression equation is not successful in prediction of C_0 with $R^2 =$ 0.314, RMSE = 1.082 and MAE = 1.075. This implies that more sophisticated methods are needed for prediction of C_i/C_v. Figure 8 presents a scatter plot of training and testing phases based on non-linear regression (Equation (8)).

Performance of GEP

The GEP model (C_i/C_v versus P/H_0 and I) was applied using different operators defined in Table 2. GEP performs better than regression type analyses, particularly for option 2 in terms of R^2 (0.984), RMSE (0.002) and MAE (0.002) in

Table 1 | Training and testing phase errors for the relative value of the discharge coefficient (C_i/C_v)

	Training phase			Testing phase		
Type	**R^2**	**RMSE**	**MAE**	**R^2**	**RMSE**	**MAE**
Equation (6)	0.998	0.004	0.003	0.996	0.007	0.009
Equation (7)	0.998	0.007	0.005	0.995	0.056	0.034
Equation (8)	0.278	0.008	0.005	0.314	1.082	1.075

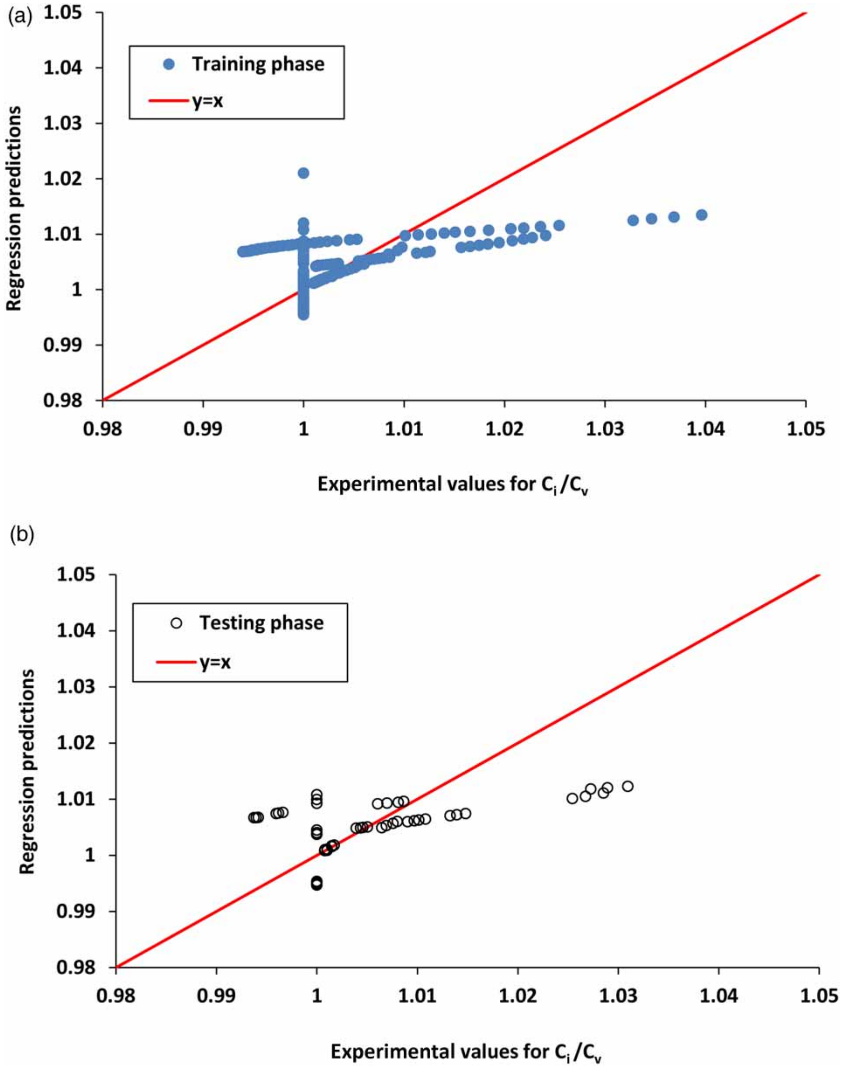

Figure 8 | Scatter plot of training (a) and testing phases (b) based on non-linear regression (Equation (8)).

Table 2 | Errors for the training and testing datasets of the relative discharge coefficient with different GEP operators

		Training phase			Testing phase		
Type	**Operator**	R^2	**RMSE**	**MAE**	R^2	**RMSE**	**MAE**
Option 1	{+, −, *, /}	0.884	0.005	0.004	0.925	0.005	0.004
Option 2	{+, −, *,/, x^2, Exp}	0.984	0.002	0.001	0.984	0.002	0.002
Option 3	{+, −, *,/, x^2, x^3, Exp, cube root}	0.903	0.004	0.004	0.927	0.004	0.003
Option 4	{+, −, *,/, x^2, x^3, Exp, cube root, 10^x, Natural logarithm}	0.857	0.005	0.004	0.935	0.004	0.003

the testing phase. The derived equation for option 2 using GEP is:

$$\frac{C_i}{C_v} = 7.387^{\left(-18.526-\frac{P}{H0}+1.212\right)} + \frac{\left(1.902+\frac{P}{H0}\right)^2}{-47.671+443.797*I} - \frac{10.670}{2.718*I*\frac{P}{H0}*(I-10.670)} \tag{9}$$

The derived GEP model in Equation (9) offers a high-order nonlinear equation that gives good accuracy with relatively low error. Performance of the GEP method is illustrated in Figure 7. According to Figure 9, GEP is well able to forecast the values of the relative discharge coefficient (C_i/C_v).

By combining Equations (6) and (8), a new graph (C_0 versus P/H_0) can be obtained with results that are shown in Figure 10. The figure is comprised of four slopes: 45, 33.69, 18.42 and 0 (vertical face) degrees. From Figure 10, it is seen that an inclination of upstream face of ogee weir increases C_0; however, the slope effect is small.

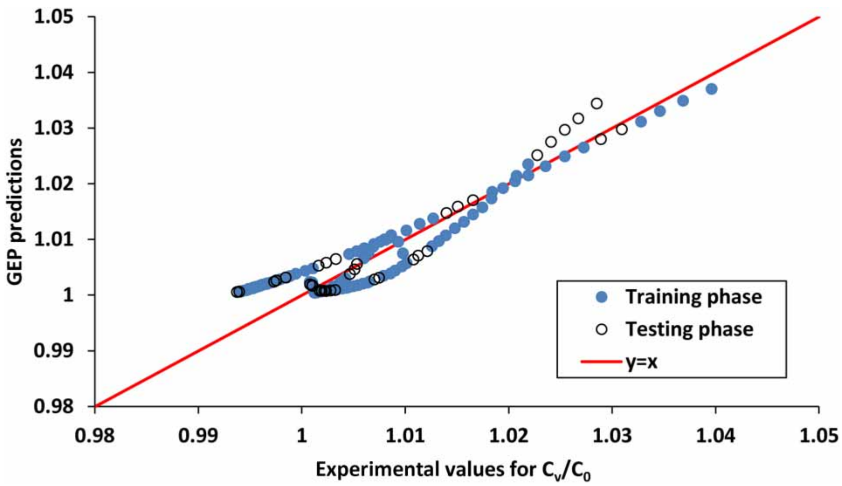

Figure 9 | Scatter plot of training and testing phases based on GEP model (Equation (9)).

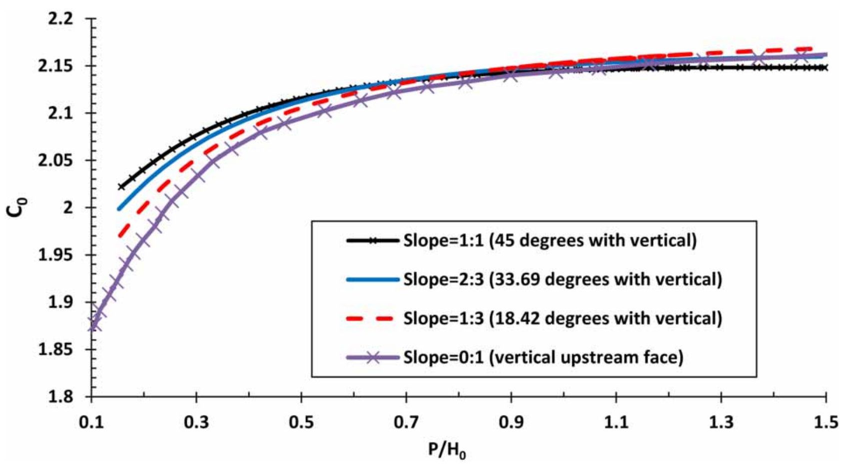

Figure 10 | Variation of discharge coefficient (C_0) with the ratio of P/H_0 for four different upstream ogee weir slopes.

Final equations for predicting C_0

Regression equation

By applying Equations (6)–(8), two equations for prediction of C_0 were obtained (Equations (12) and (13)). A scatter plot for Equation (11) and errors for both the testing and training data of the discharge coefficient (C_0) related with two regression models are presented in Figure 11 and Table 3, respectively. Equation (11) has better performance ($R^2 = 0.702$, RMSE $= 0.792$ and MAE $= 0.803$).

$$C_0 = 0.001(I) + 2.119\left(\frac{He}{H0}\right)^{0.128}\left(\frac{P}{H0}\right)^{0.041} \quad (10)$$

Table 3 | Estimation of the errors for the training and testing datasets of the C_0 with different regression models

	Training phase			Testing phase		
Type	R^2	RMSE	MAE	R^2	RMSE	MAE
Equation (10)	0.602	0.992	0.959	0.587	0.410	0.705
Equation (11)	0.724	0.784	0.959	0.702	0.792	0.803

$$C_0 = 0.001(I) + 5.050\left(\frac{He}{H0}\right)^{0.049} - 2.930\left(\frac{P}{H0}\right)^{-0.025} \quad (11)$$

GEP equation

The results from the GEP analysis for C_0 are given in Equation (12) and prediction errors are given in Table 4.

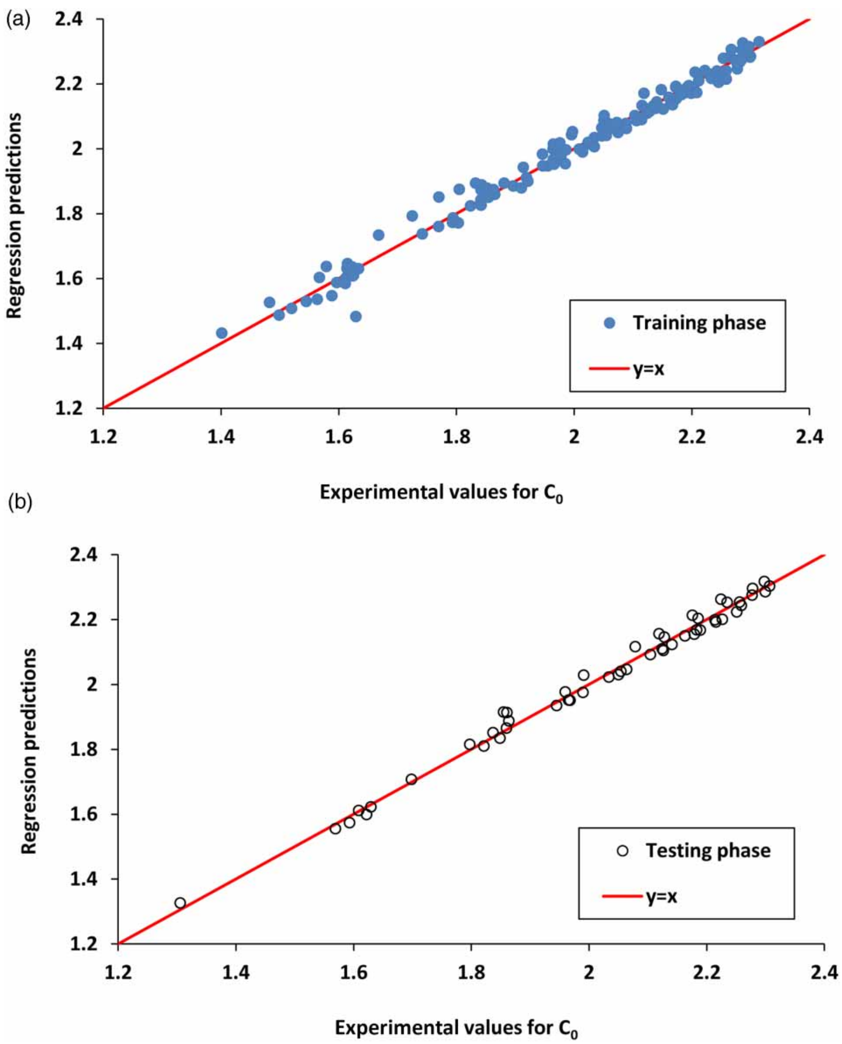

Figure 11 | Comparison between experimental data and regression model (Equation (10)): (a) training phase, (b) testing phase.

Table 4 | Estimation of the errors for testing and training of the relative discharge coefficient with different GEP operators

		Training phase			Testing phase		
Type	**Operator**	R^2	**RMSE**	**MAE**	R^2	**RMSE**	**MAE**
Option 1	{+, −, *, /}	0.994	0.023	0.016	0.987	0.033	0.021
Option 2	{+, −, *,/, x^2, Exp}	0.989	0.032	0.027	0.985	0.035	0.028
Option 3	{+, −, *,/, x^2, x^3, Exp, cube root}	0.998	0.016	0.012	0.995	0.021	0.015
Option 4	{+, −, *,/, x^2, x^3, Exp, cube root, 10^x, Natural logarithm}	0.978	0.047	0.042	0.977	0.047	0.040

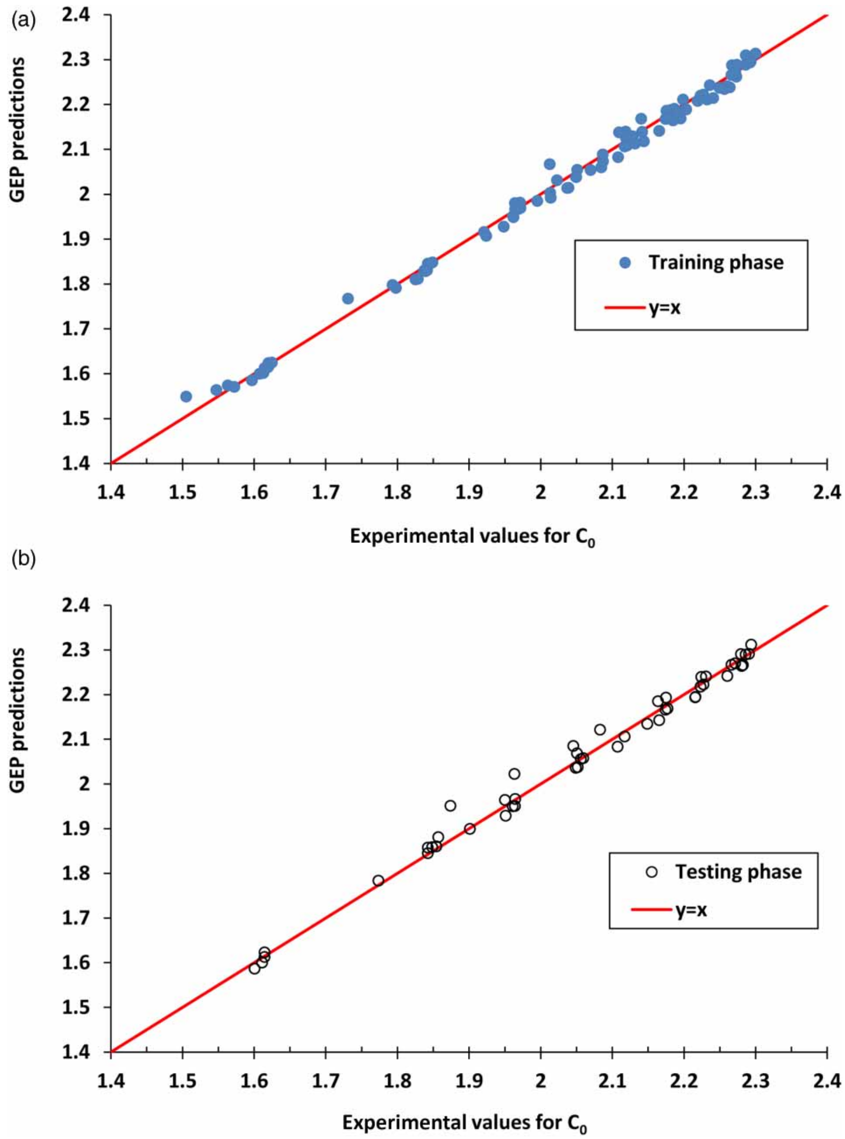

Figure 12 | Comparison between experimental data and GEP, the model (option 3): (a) for the training phase, (b) for the testing phase.

Table 5 | Prediction of the discharge coefficient for an ogee weir with inclined in upstream face (*I*) and with head (H_e) that differs from the design head (H_O)

L (m)	P (m)	H_0 (m)	P/H_0	I (Degrees)	H_e (m)	H_e/H_0	C_0	Q (m^3/s)	$Q_{inclined}/Q_{vertical}$
10	5	2	2.5	0	0.2	0.10	1.785	1.597	1
10	5	2	2.5	0	0.4	0.20	1.862	4.710	1
10	5	2	2.5	0	0.6	0.30	1.932	8.980	1
10	5	2	2.5	0	0.8	0.40	1.997	14.289	1
10	5	2	2.5	0	1	0.50	2.056	20.556	1
10	5	2	2.5	0	1.2	0.60	2.108	27.715	1
10	5	2	2.5	0	1.4	0.70	2.155	35.700	1
10	5	2	2.5	0	1.6	0.80	2.196	44.444	1
10	5	2	2.5	0	1.8	0.90	2.231	53.874	1
10	5	2	2.5	0	2	1.00	2.260	63.917	1
10	5	2	2.5	0	2.2	1.10	2.283	74.490	1
10	5	2	2.5	0	2.4	1.20	2.300	85.509	1
10	5	2	2.5	0	2.6	1.30	2.311	96.882	1
10	5	2	2.5	0	2.8	1.40	2.316	108.513	1
10	5	2	2.5	0	3	1.50	2.315	120.301	1
10	5	2	2.5	0	3.2	1.60	2.308	132.142	1
10	5	2	2.5	25	0.2	0.10	1.790	1.601	1.003
10	5	2	2.5	25	0.4	0.20	1.872	4.735	1.005
10	5	2	2.5	25	0.6	0.30	1.948	9.051	1.008
10	5	2	2.5	25	0.8	0.40	2.017	14.435	1.010
10	5	2	2.5	25	1	0.50	2.081	20.811	1.012
10	5	2	2.5	25	1.2	0.60	2.139	28.117	1.015
10	5	2	2.5	25	1.4	0.70	2.191	36.291	1.017
10	5	2	2.5	25	1.6	0.80	2.237	45.269	1.019
10	5	2	2.5	25	1.8	0.90	2.277	54.983	1.021
10	5	2	2.5	25	2	1.00	2.311	65.359	1.023
10	5	2	2.5	25	2.2	1.10	2.339	76.320	1.025
10	5	2	2.5	25	2.4	1.20	2.361	87.784	1.027
10	5	2	2.5	25	2.6	1.30	2.377	99.660	1.029
10	5	2	2.5	25	2.8	1.40	2.387	111.857	1.031
10	5	2	2.5	25	3	1.50	2.392	124.275	1.033
10	5	2	2.5	25	3.2	1.60	2.390	136.811	1.035
10	5	2	2.5	45	0.2	0.10	1.794	1.605	1.005
10	5	2	2.5	45	0.4	0.20	1.880	4.757	1.010
10	5	2	2.5	45	0.6	0.30	1.960	9.111	1.015
10	5	2	2.5	45	0.8	0.40	2.034	14.557	1.019
10	5	2	2.5	45	1	0.50	2.102	21.024	1.023
10	5	2	2.5	45	1.2	0.60	2.165	28.454	1.027
10	5	2	2.5	45	1.4	0.70	2.221	36.786	1.030
10	5	2	2.5	45	1.6	0.80	2.271	45.961	1.034

(*continued*)

Table 5 | continued

L (m)	P (m)	H_0 (m)	P/H_0	I (Degrees)	H_e (m)	H_e/H_0	C_0	Q (m^3/s)	$Q_{inclined}/Q_{vertical}$
10	5	2	2.5	45	1.8	0.90	2.315	55.911	1.038
10	5	2	2.5	45	2	1.00	2.354	66.567	1.041
10	5	2	2.5	45	2.2	1.10	2.386	77.854	1.045
10	5	2	2.5	45	2.4	1.20	2.412	89.689	1.049
10	5	2	2.5	45	2.6	1.30	2.433	101.988	1.053
10	5	2	2.5	45	2.8	1.40	2.447	114.659	1.057
10	5	2	2.5	45	3	1.50	2.456	127.605	1.061
10	5	2	2.5	45	3.2	1.60	2.458	140.724	1.065
10	5	2	2.5	60	0.2	0.10	1.798	1.608	1.007
10	5	2	2.5	60	0.4	0.20	1.887	4.774	1.014
10	5	2	2.5	60	0.6	0.30	1.970	9.157	1.020
10	5	2	2.5	60	0.8	0.40	2.048	14.652	1.025
10	5	2	2.5	60	1	0.50	2.119	21.190	1.031
10	5	2	2.5	60	1.2	0.60	2.184	28.716	1.036
10	5	2	2.5	60	1.4	0.70	2.244	37.171	1.041
10	5	2	2.5	60	1.6	0.80	2.298	46.498	1.046
10	5	2	2.5	60	1.8	0.90	2.345	56.633	1.051
10	5	2	2.5	60	2	1.00	2.387	67.507	1.056
10	5	2	2.5	60	2.2	1.10	2.422	79.046	1.061
10	5	2	2.5	60	2.4	1.20	2.452	91.171	1.066
10	5	2	2.5	60	2.6	1.30	2.476	103.798	1.071
10	5	2	2.5	60	2.8	1.40	2.494	116.837	1.077
10	5	2	2.5	60	3	1.50	2.506	130.193	1.082
10	5	2	2.5	60	3.2	1.60	2.511	143.765	1.088

The performance errors from the preferred GEP were: $R^2 = 0.995$, $RMSE = 0.021$ and $MAE = 0.015$ for the testing phase.

Figure 12 shows comparison between experimental data and the GEP model (option 3) for training and testing phases.

$$C_0 = \frac{I + 2\left(\frac{P}{H_0}\right) + 1.697}{697.362 * 10^{P/H0}} + \frac{\frac{P}{H0}}{-3.705\left(\frac{He}{H0}\right) - \frac{I + 2.161}{P/H0}} + \frac{I - 4.453}{I - 5.002 + e^{-7.140}} \tag{12}$$

Application example

To demonstrate a practical application of the discussed method, it is used to calculate the discharge coefficient (C_0) of an ogee weir due to the existence of an upstream sloping face and a discharge over it that differs from the design discharge with the following characteristics:

The design discharge for the weir (with a vertical face) is 63.917 m^3/s. The overflow dam spillway height is 5 m. The length of the overflow spillway is 10 m and the design head is 2 m. The upstream face of the weir is inclined at 0, 25, 45 and 60 degrees. The values of C_0 are computed using Equation (12) and Q is computed with Equation (1). The results are presented in Table 5.

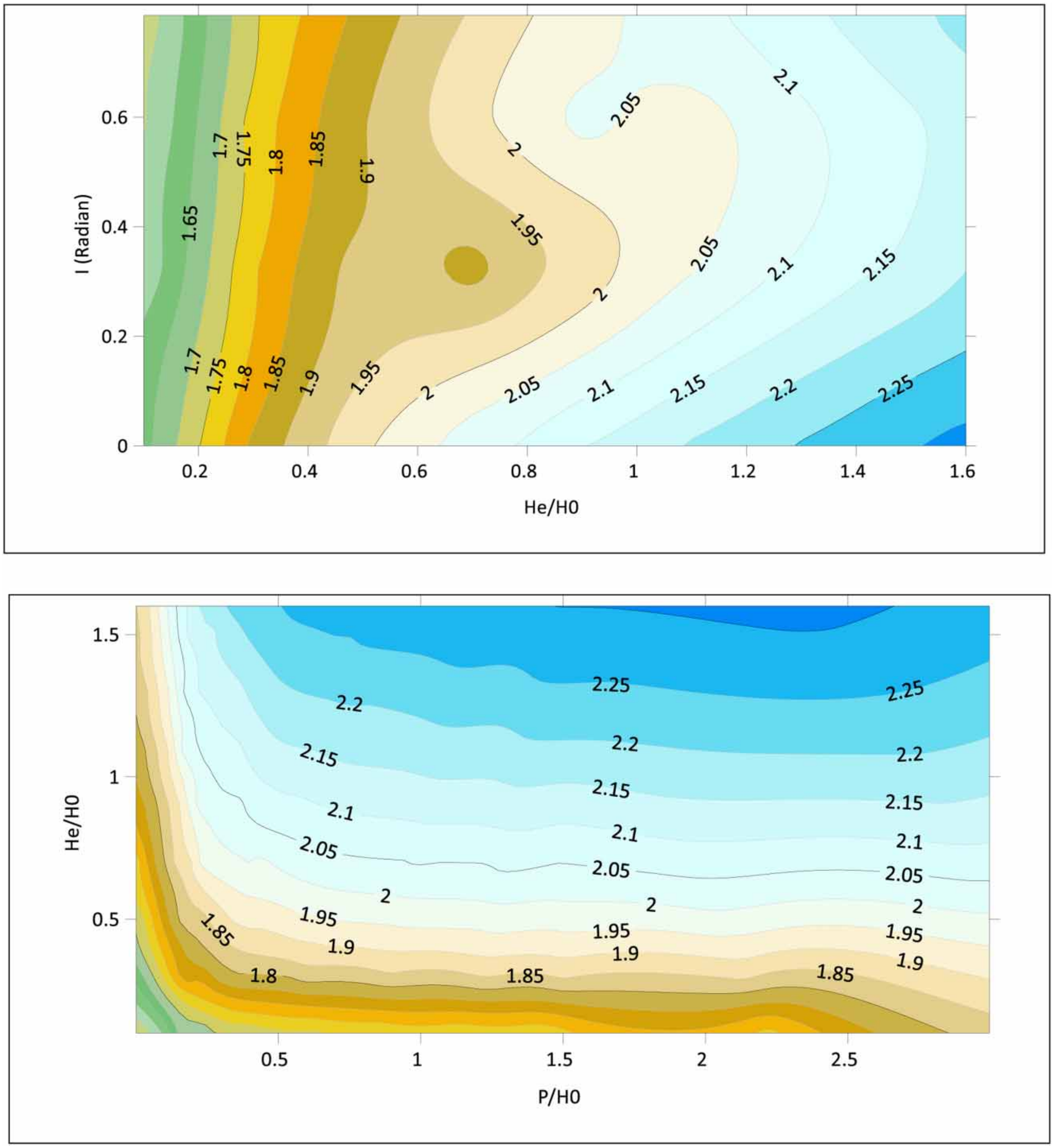

Figure 13 | Contours for estimation of C_d.

Table 5 shows that the inclination of the upstream face causes an increase in the discharge coefficient (C_0). This can be seen from comparison of $I = 0$ degrees with $I = 25$, 45 and 60 degrees. The last column in Table 5 demonstrates that the ratio of $Q_{\text{inclined}}/Q_{\text{vertical}}$ exceeds 1 for I values greater than 0.

Figure 13 provides additional information about the relation among independent variables (P/H_e and H_e/H_0) against dependent variable C_d. These contours represent values for C_d estimation based on the 202 data points.

The relation between H_e/H_0 and I shows that for a constant value of H_e/H_0, increases in I causes decreases in C_d. In addition, for a constant value for I, increasing H_e/H_0 results in increases in C_d.

The relationship between P/H_0 and H_e/H_0 shows that for a constant value for P/H_0, increasing H_e/H_0 causes an increase in C_d. For a constant value of H_e/H_0, increasing P/H_0 has no effect on C_d except for $P/H_0 < 0.25$.

CONCLUSIONS

This study creates a predictive model that accurately quantifies the discharge coefficient (C_0) from ogee weirs with upstream sloping faces and with discharge other than the design discharge. Regression analysis and GEP were carried out. Parameters such as weir upstream face inclination (I), ratio of P/H_0 that reflects weir height over the design head, and the ratio of H_e/H_0 are the input variables, while discharge coefficient (C_0) was an output. The GEP technique was more capable than regression analysis in predicting C_0. Performance errors from the preferred GEP analysis were $R^2 = 0.995$, RMSE = 0.021 and MAE = 0.015, while for a non-linear regression equation they were: $R^2 = 0.702$, RMSE = 0.792 and MAE = 0.803. Finally, examples were presented to show the application of the suggested equations. These examples accounted for 0, 25, 45 and 60 degree upstream weir slopes and included different ratios of actual heads (H_e/H_0).

FUNDING

There is no funding for this article.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

DECLARATIONS OF INTEREST

None.

ACKNOWLEDGEMENTS

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

ETHICAL APPROVAL

This article does not contain any studies with human participants or animals performed by any of the authors.

SUPPLEMENTARY MATERIAL

The Supplementary Material for this paper is available online at https://dx.doi.org/10.2166/ws.2020.064.

REFERENCES

Akbari, M., Salmasi, F., Arvanaghi, H., Karbasi, M. & Farsadizadeh, D. 2019 Application of Gaussian process regression model to predict discharge coefficient of Gated Piano Key Weir. *Water Resources Management* **33** (11), 3929–3947. https://doi.org/10.1007/s11269-019-02343-3.

Al-Khatib I, A. & Gogus, M. 2014 Prediction models for discharge estimation in rectangular compound broad-crested weirs. *Flow Measurement and Instrumentation* **36**, 1–8. doi.org/10.1016/j.flowmeasinst.2014.01.001.

Bradley, J. N. 1945 *Study of Flow Characteristics, Discharge and Pressures Relative to Submerged Dams. Hydraulic Laboratory Rep. No. 182*. US Bureau of Reclamation, Denver.

CurveExpert Professional (2012) User guide.

Ferreira, C. 2001a Gene expression programming in problem solving. In: *6th Online World Conference on Soft Computing in Industrial Applications (Invited Tutorial)*.

Ferreira, C. 2001b Gene expression programming: a new adaptive algorithm for solving problems. *Complex Systems* **13** (2), 87–129.

GeneXproTools (2011) Version 4.0. User manual.

Guven, A., Hassan, M. & Sabir, S. 2013 Experimental investigation on discharge coefficient for a combined broad crested weir-box culvert structure. *Journal of Hydrology* **500** (13), 97–103. doi:10.1016/j.jhydrol.2013.07.021.

Kim, D. G. & Park, J. H. 2005 Analysis of flow structure over ogee-spillway in consideration of scale and roughness effects by using CFD model. *KSCE Journal of Civil Engineering* **9**, 161–169. https://doi.org/10.1007/BF02829067.

Madadi, M. R., Hosseinzadeh Dalir, A. & Farsadizadeh, D. 2014 Investigation of flow characteristics above trapezoidal broad-crested weirs. *Flow Measurement and Instrumentation* **38**, 139–148. doi:10.1016/j.flowmeasinst.2014.05.014.

Mohamed, H. 2010 Flow over gabion weirs. *Journal of Irrigation and Drainage Engineering* **136** (8), 573–577.

Moody, M. L. 1947 An approximate formula for pipe friction factors. *Transactions of the ASME* **69**, 1005–1009.

Salmasi, F. 2018 Effect of downstream apron elevation and downstream submergence in discharge coefficient of ogee weir. *ISH Journal of Hydraulic Engineering.* doi:10.1080/09715010.2018.1556125.

Salmasi, F. 2019 Discussion of 'experimental and CFD analysis of circular labyrinth weirs'. *Journal of Irrigation and Drainage Engineering* **145** (4), 07019001. doi: 10.1061/(ASCE)IR.1943-4774.0001301.

Salmasi, F. & Abraham, J. 2020 Discussion of 'Hydrodynamics of rectangular broad-crested Porous Weirs' by Akbar Safarzadeh and Seyed Hossein Mohajeri. *Journal of Irrigation and Drainage Engineering* **146** (4), 07020003. https://doi.org/10.1061/(ASCE)IR.1943-4774.0001450.

Salmasi, F., Khatibi, R. & Ghorbani, M. A. 2012 A study of friction factor formulation in pipes using artificial intelligence techniques and explicit equations. *Turkish Journal of Engineering and Environmental Science, TUBITAK* **36**, 121–138. doi:10.3906/muh-1008-30.

Salmasi, F., Nouri, M. & Abraham, J. 2019 Laboratory study of the effect of sills on radial gate discharge coefficient. *KSCE Journal of Civil Engineering.* **23** (5), 2117–2125. doi: 10.1007/s12205-019-1114-y.

Savage, B. M. & Johnson, M. C. 2001 Flow over ogee spillway: physical and numerical model case study. *Journal of Hydraulic Engineering* **127** (8), 640–649. https://doi.org/10.1061/(ASCE)0733-9429(2001)127:8(640).

SPSS (2013) Statistical package for social science (SPSS) software version 22.

Tullis, B. P. 2011 Behavior of submerged ogee crest weir discharge coefficients. *Journal of Irrigation and Drainage Engineering.* **137** (10), 677–681. doi:10.1061/(ASCE)IR.1943-4774.0000330.

Tullis, B. P. & Neilson, J. 2008 Performance of submerged ogee-crest weir head-discharge relationships. *Journal of Hydraulic Engineering* **134** (4), 486–491. doi:10.1061/(ASCE)0733-9429(2008)134:4(486).

US Bureau of Reclamation, USBR 1987 *Design of Small Dams*. US Government Printing Office, Washington, DC.

First received 3 May 2019; accepted in revised form 31 March 2020. Available online 17 April 2020