Approximations of the Darcy–Weisbach friction factor in a vertical pipe with full flow regime

Zhang Zeyu, Chai Junrui, Li Zhanbin, Xu Zengguang and Li Peng

ABSTRACT

The discharge in a full flow regime represents the discharge capacity of a vertical pipe, and the Darcy–Weisbach friction factor (λ) is an important variable to calculate discharge. Since all existing equations for λ contain the Reynolds number (R_e), it is problematic if the velocity is unknown. In this study, the performance of existing equations collected from studies on vertical pipes is assessed, and an approximation for the λ of vertical pipes in the full flow regime, without R_e, is proposed. The performance of the Brkić and Praks equation is the best, with a maximum relative error (MRE) of 0.003% (extremely accurate). The MRE of the new approximation is 0.43%, and its assessment level is very accurate. This work is expected to provide a reference for the design and investigation of the drainage of vertical pipes.

Key words | Colebrook equation, friction factor, rough pipe, smooth pipe, vertical pipe

Zhang Zeyu
Chai Junrui (corresponding author)
Li Zhanbin
Xu Zengguang
Li Peng
State Key Laboratory of Eco-hydraulics in Northwest Arid Region of China,
Xi'an University of Technology,
Xi'an 710048,
China
E-mail: *jrchai@xaut.edu.cn*

Chai Junrui
School of Civil Engineering,
Xijing University,
Xi'an 710123,
China

Li Zhanbin
State Key Laboratory of Soil Erosion and Dry-land Farming on the Loess Plateau,
Institute of Soil and Water Conservation, Chinese Academy of Sciences and Ministry of Water Resources,
Yangling, Shaanxi 712100,
China

NOTATION

The following symbols are used in this paper:

- C discharge coefficient (–);
- d diameter of the vertical pipe (m);
- d_b diameter of the barrel (m);
- g gravitational acceleration (m s^{-2});
- h head above the crest of the vertical pipe (m);
- l length of the vertical pipe (m);
- l_b length of the barrel (m);
- P projection length of the vertical pipe over the tank floor (m);
- Q discharge (m^3 s^{-1});
- R_e Reynolds number (–);
- α angle between the horizontal plane and centerline of the barrel (°);
- γ viscosity (m^2 s^{-1});
- ε/d relative roughness of pipe (–);
- ζ_e entrance loss coefficient (–);
- λ Darcy–Weisbach friction factor (–);
- v mean velocity (m s^{-1}).

INTRODUCTION

A vertical pipe, which can be distinguished as a vertical drain or an overflow pipe depending on whether the pipe extends above the tank floor (Kalinske 1940), is widely used in different types of drainage systems, such as roof rain leaders (Padulano & Del Giudice 2018), manholes (Banisoltan *et al.* 2015) and tank drains. In addition, such pipes are used in spillways of dams after performing certain geometric optimizations, such as in morning glory spillways (Leopardi 2014). In general, existing studies define three flow regimes of a vertical pipe, which can also be subdivided further (Banisoltan *et al.* 2017; Padulano & Del Giudice

doi: 10.2166/ws.2020.048

2018). The three flow regimes, with the head varying from low to high, are weir-like, transition and full flow regimes. The discharge in a full flow regime corresponds to the conveying capacity of a vertical pipe, and it is an essential parameter for a drainage device. In addition, the full flow regime is the only regime in which the discharge coefficient can be derived using the energy conservation equation.

A vertical pipe is considered to be fully filled, and the swirl on the water surface is considered to be infinitesimal or even nonexistent, in the full flow regime, as shown in Figure 1. The discharge of full flow regimes can be defined as $Q = C{*}\pi d^2/4{*}\sqrt{2g(h+l)}$; here, the discharge coefficient C can be calculated as $(1+\lambda{*}\,l/d+\zeta_e)^{-0.5}$, where λ is the Darcy–Weisbach friction factor and ζ_e is the entrance loss coefficient.

The Darcy–Weisbach friction factor (λ) has been investigated in several studies, and is believed to be affected by the material and size of pipe, and the velocity of the flow in the pipe as well (Fanning 1877). During the development of solving λ, the relative roughness (ε/d) and the Reynolds number (R_e) are introduced to describe the three characteristics; and two important implicit equations, the Colebrook equation (a function of ε/d and R_e) and the Nikuradse–Prandtl–von Karman (NPK) equation (a function of R_e), are proposed for solving λ of the rough pipes and smooth pipes, respectively. Furthermore, explicit equations abound that have been developed based on them. For example, Samadianfard (2012) proposed an explicit solution of λ based on the Colebrook equation with gene expression programming analysis; Brkić & Praks (2018) derived an accurate explicit approximation of the Colebrook equation with the Wright ω-Function (a shifted Lambert W-function);

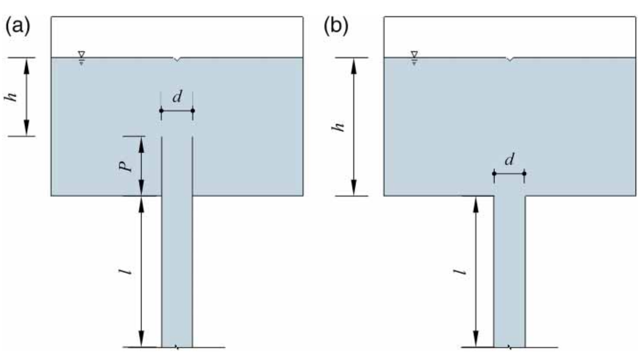

Figure 1 | Schematic of the full flow regime in (a) an overflow pipe, (b) a vertical drain.

Brkić (2011b) and Heydari et al. (2015) took the complexity of equations into consideration for the comparison of explicit equations; analogously, Li et al. (2011) compared the computation time of explicit solutions and proposed an explicit determination of λ based on the NPK equation with a ternary cubic polynomial of $\ln R_e$ for the smooth pipe case. The details of the explicit equations are collected and discussed in the third section of this paper. However, since $R_e = vd/\gamma$, it is problematic if the flow velocity in the vertical pipe (v) is unknown, such as in the design stage. Alazba et al. (2012), in their study on the friction head-loss of center-pivot irrigation machines, proposed a simple equation for λ with a constant velocity value based on the field data.

In this study, the accuracy of the existing equations for λ in the case of the full flow regime in a vertical pipe is assessed, and a new equation for λ that does not involve the variable v is proposed based on the data from Anwar (1965), Padulano et al. (2013), Padulano et al. (2015) and Banisoltan et al. (2017).

STATE-OF-THE-ART REVIEW OF FULL FLOW IN A VERTICAL PIPE

In this study, the literature pertaining to the flow in a vertical pipe was investigated, and the details of the experiments performed in these studies are listed in Table 1; the sketch of the inflow condition is shown in Figure 2. Figure 3 shows the discharge coefficient C as a function of the non-dimensional water head h/d; the corresponding values of ε/d and R_e are shown in Figure 4. The value of R_e recorded in the literature is within the range of $(1\times10^5,\ 6\times10^5)$, and that of ε/d is within $(1\times10^{-5},\ 1.5\times10^{-4})$.

SOLUTIONS FOR THE DARCY–WEISBACH FRICTION FACTOR

Friction factor for turbulent flow in rough pipes

For turbulent flow in rough pipes, the Colebrook equation is regarded as a transcendental expression of λ (Brkić 2011b). Since the Colebrook equation is an implicit function,

Table 1 | Details of experiments reported in the literature

		Vertical pipe			Barrel		
Source	**Inflow condition**	**Diameter d (m)**	**Length l (m)**	**Projected distance P (m)**	**Angle α (°)**	**Diameter d_b (m)**	**Length l_b (m)**
Anwar (1965)	Radial	0.0663	0.609	0.15	–		
		0.1016					
		0.0384					
Padulano *et al.* (2013); Padulano *et al.* (2015)	Unilateral	0.07	1.5	0	–		
		0.1	1				
Banisoltan *et al.* (2017)	Radial	0.076	1.219	0	–		
Humphreys *et al.* (1970)	Unilateral	0.0758	1.78	0.105	0	0.053	2.79
		0.1265	0.38	0.152	17.5	0.076	7.62
				0			
		0.2015	0.61	0.379			
Zhang (2017)	Unilateral	0.1	0.96	0.19	0.573	0.080	7.40

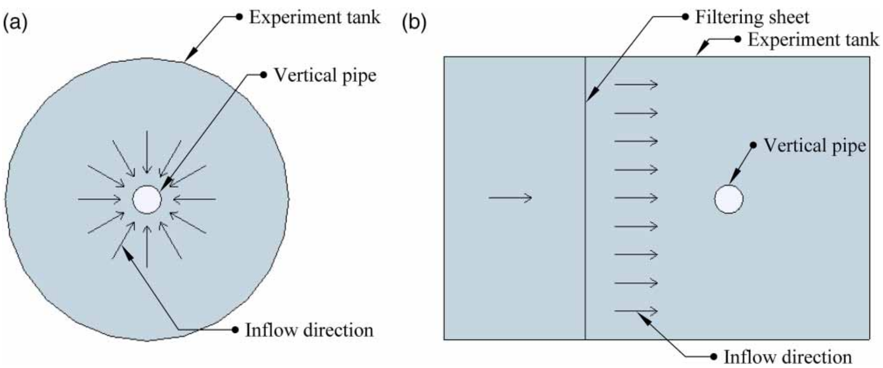

Figure 2 | Sketch of inflow condition: (a) radial flow; (b) unilateral flow.

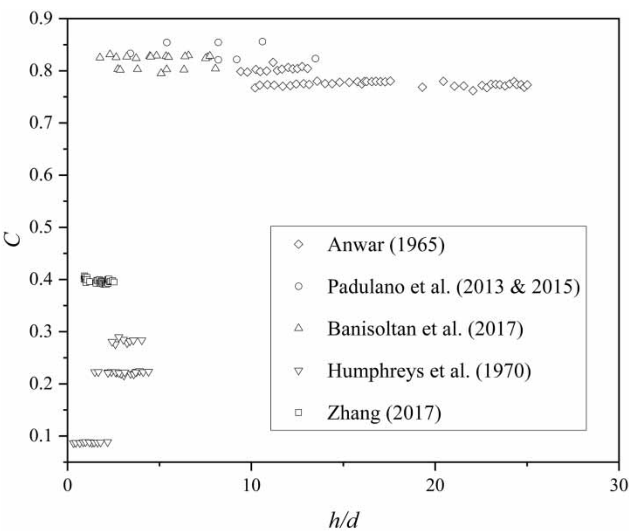

Figure 3 | Discharge coefficient C as a function of the non-dimensional water head h/d.

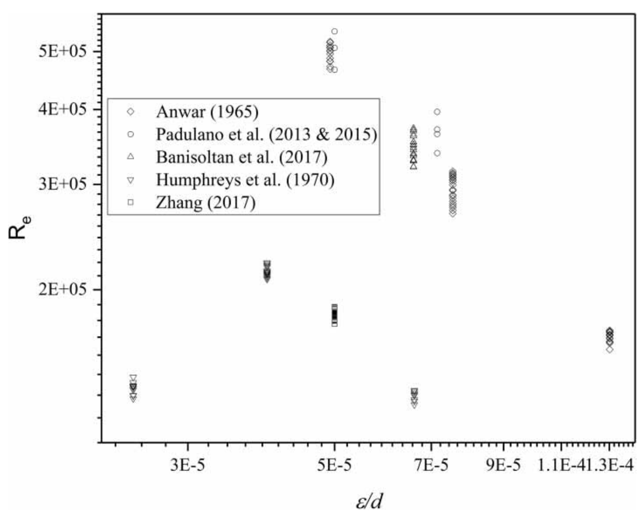

Figure 4 | Ranges of the relative roughness and Reynolds number of the collected data.

many researchers have proposed explicit approximations to avoid the iteration solution. Among these, the most popular approximations were collected, as presented in Table 2. The friction factor (λ) of turbulent flow in rough pipes is believed to be affected by R_e and ε/D. In general, the relationship among these parameters can be classified into the following three categories: logarithmic, power (No. 1, No. 2, No. 4, No. 19 and No. 32 in Table 2) and the combination of the fronts (No. 8 in Table 2).

Friction factor for turbulent flow in smooth pipes

For a smooth pipe, the NPK equation is used to evaluate the accuracy of the explicit approximate equations. The most popular approximations were collected, as presented in Table 3. Since the NPK equation can be regarded as a particular case of the Colebrook equation in which the roughness is completely absent, Brkić (2011a) believed that an approximation of the NPK equation in a suitable form can be transformed into an approximation of the Colebrook equation, i.e. $\lambda = (-2\log x)^{-2} \to \lambda = \{-2\log[x + \varepsilon/(3.71d)]\}^{-2}$. The approximations with matching forms were thus transformed and are presented in Table 3.

ANALYSIS AND RESULT

Assessment of accuracy of the existing approximations

The Darcy–Weisbach friction factor (λ) corresponding to the data from Anwar (1965), Padulano et al. (2013), Padulano et al. (2015) and Banisoltan et al. (2017) was calculated using the Colebrook equation and the approximations mentioned previously, and the maximum relative errors (MRE) of the approximations compared with the Colebrook equation were determined, as shown in Figure 5. Offor & Alabi (2016) classified the accuracy of approximations considering the MRE, and the threshold value was considered to be 5%. It is thus inadvisable to accept an approximation that has an MRE greater than 5%, and approximations with an MRE of up to 0.14% and 0.5% are assessed as extremely accurate and very accurate, respectively. Accordingly, 34 approximation equations exist for rough pipes, which are acceptable among the 40 approximations mentioned (the MRE of the equation of Rao and Kumar, listed as No. 23 in Table 2, is 26.88%, and it is not shown in Figure 5 considering the scale of the chart). Among these 34 equations, 24 approximations were assessed as very accurate and three were assessed as extremely accurate. The MREs of the extremely accurate approximations were 0.003%, 0.010%, and 0.014% for the Brkić and Praks (No. 39 in Table 2), Biberg (No. 37 in Table 2) and Serghides I (No. 17 in Table 2) approximations, respectively.

For the equations transformed for rough pipes, the MREs of all the approximations decrease after transformation, as observed from Figure 5. The values of all MREs are less than 5%, which indicates that these values are acceptable. Among the considered approximations, four can be judged as very accurate. Thus, the transformed equations from the approximations for smooth pipe can be suitably employed to calculate the λ value for a rough pipe.

Proposed equation for the friction factor not including R_e

To propose a new approximation for λ, the data from Anwar (1965), Padulano et al. (2013), Padulano et al. (2015) and Banisoltan et al. (2017) were employed. According to the brief review of existing approximations for the λ of a pipe, λ can be represented using logarithmic and power law equations. Since a logarithmic equation can be approximated with a power law form and considering the convenience of further analysis, the power law form was selected for deriving the new approximation, i.e., $y = ax_1^b + cx_2^d + \cdots + m$. The variables ε/d, R_e, $\varepsilon/d\cdot R_e$, $(\varepsilon/d)/R_e$ and their logarithms were used in the approximations mentioned above. In the full flow regime of a vertical pipe, the flow velocity is determined by $h + l$ (for the overflow pipe, l should be replaced with $P + l$); accordingly, $(h + l)/d$ was used to replace R_e in the new approximation equation of λ. In particular, the variables ε/d, $(h + l)/d$, $\varepsilon/d \cdot (h + l)/d$, $(\varepsilon/d)/[(h + l)/d]$, $\ln(\varepsilon/D)$, $\ln[(h + l)/d]$, $\ln[\varepsilon/d \cdot (h + l)/d]$ and $\ln\{(\varepsilon/d)/[(h + l)/d]\}$ were used to establish the new equation. The resulting expression can be defined as Equation (1); this approximation had an MRE of 0.43%, which corresponds to an assessment level of very accurate. The comparison of the λ values calculated using the

Table 2 | Summary of the approximation equations for the friction factor of rough pipes

No.	Author and source	Year	Equation for rough pipe
1	Moody (Brkić 2011b)	1947	$\lambda = 0.0055\left[1+\left(2\times 10^4\frac{\varepsilon}{d}+\frac{10^6}{R_e}\right)^{1/3}\right]$
2	Altshul (Olivares *et al.* 2019)	1952	$\lambda = 0.11\left(\frac{\varepsilon}{d}+\frac{68}{R_e}\right)^{0.25}$
3	Altshul II (Olivares *et al.* 2019)	1952	$\frac{1}{\sqrt{\lambda}} = 1.8\log\frac{R_e}{0.135R_e\,\varepsilon/d+6.5}$
4	Wood (Brkić 2011b)	1966	$\lambda = 0.094(\varepsilon/d)^{0.225}+0.53\varepsilon/d+88(\varepsilon/d)^{0.44}R_e^{-V}$ where $V = 1.62(\varepsilon/d)^{0.134}$
5	Eck (Brkić 2011b)	1973	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.715d}+\frac{1.5}{R_e}\right]$
6	Jain (Brkić 2011b)	1976	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.715d}+\left(\frac{6.943}{R_e}\right)^{0.9}\right]$
7	Swamee and Jain (Olivares *et al.* 2019)	1976	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.7d}+\frac{5.74}{R_e^{0.9}}\right)$
8	Churchill (Brkić 2011b)	1977	$\frac{1}{\sqrt{\lambda}} = 8\left[\left(\frac{8}{R_e}\right)^{12}+\frac{1}{(C_1+C_2)^{1.5}}\right]^{\frac{1}{12}}$ where $C_1 = \left[2.457\ln\frac{1}{(7/R_e)^{0.9}+0.27\varepsilon/d}\right]^{16}$ $C_2 = \left(\frac{37,530}{R_e}\right)^{16}$
9	Chen (Brkić 2011b)	1979	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.7065d}-\frac{5.0452}{R_e}\log\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{d}\right)^{1.1098}+\frac{5.8506}{R_e^{0.8981}}\right)\right]$
10	Round (Olivares *et al.* 2019)	1980	$\frac{1}{\sqrt{\lambda}} = -1.8\log\left(\frac{0.27\varepsilon}{d}+\frac{6.5}{R_e}\right)$
11	Shacham (Zigrang & Sylvester 1985)	1980	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.7d}+\frac{5.02}{R_e}\log\left(\frac{\varepsilon}{3.7d}+\frac{14.5}{R_e}\right)\right]$
12	Shacham II (Olivares *et al.* 2019)	1980	$\frac{1}{\sqrt{\lambda}} = \frac{X(1-\ln X)-\frac{\varepsilon}{3.7d}}{1.15129X+2.51/R_e}$ where $X = \frac{\varepsilon}{3.7d}-\frac{5.02}{R_e}\log\left(\frac{\varepsilon}{3.7d}+\frac{14.5}{R_e}\right)$
13	Barr (Olivares *et al.* 2019)	1981	$\frac{1}{\sqrt{\lambda}} = -2\log\left\{\frac{\varepsilon}{3.7d}+\frac{4.518\log\frac{R_e}{7}}{R_e\left[1+\frac{R_e^{0.52}}{29}\left(\frac{\varepsilon}{d}\right)^{0.7}\right]}\right\}$

(*continued*)

Table 2 | continued

No.	Author and source	Year	Equation for rough pipe
14	Pavlov (Olivares *et al.* 2019)	1981	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.7d} + \left(\frac{6.81}{\mathrm{R_e}}\right)^{0.9}\right]$
15	Zigrang–Sylvester (Zigrang & Sylvester 1982)	1982	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{\varepsilon}{3.7d} - \frac{5.02}{\mathrm{R_e}}\log\left(\frac{\varepsilon}{3.7d} - \frac{5.02}{\mathrm{R_e}}\log\left(\frac{\varepsilon}{3.7d} + \frac{13}{\mathrm{R_e}}\right)\right)\right]$
16	S. E. Haaland (Haaland 1983)	1983	$\frac{1}{\sqrt{\lambda}} = -1.8\log\left[\left(\frac{\varepsilon}{3.7d}\right)^{1.11} + \frac{6.9}{\mathrm{R_e}}\right]$
17	Serghides I (Brkić 2011b)	1984	$\frac{1}{\sqrt{\lambda}} = S_1 - \frac{(S_1 - S_2)^2}{S_3 - 2S_2 + S_1}$ where $S_1 = -2\log\left(\frac{\varepsilon}{3.7d} + \frac{12}{\mathrm{R_e}}\right)$ $S_2 = -2\log\left(\frac{\varepsilon}{3.7d} + \frac{2.51S_1}{\mathrm{R_e}}\right)$ $S_3 = -2\log\left(\frac{\varepsilon}{3.7d} + \frac{2.51S_2}{\mathrm{R_e}}\right)$
18	Serghides II (Brkić 2011b)	1984	$\frac{1}{\sqrt{\lambda}} = 4.781 - \frac{(S_1 - 4.781)^2}{S_2 - 2S_1 + 4.781}$ where $S_1 = -2\log\left(\frac{\varepsilon}{3.7d} + \frac{12}{\mathrm{R_e}}\right)$ $S_2 = -2\log\left(\frac{\varepsilon}{3.7d} + \frac{2.51S_1}{\mathrm{R_e}}\right)$
19	Tsal (Asker *et al.* 2014)	1989	$\lambda = \begin{cases} C & \text{if } C \geq 0.018 \\ 0.0028 + 0.85C & \text{if } C < 0.018 \end{cases}$ where $C = 0.11(68/\mathrm{R_e} + \varepsilon/d)^{0.25}$
20	Manadilli (Olivares *et al.* 2019)	1997	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.7d} + \frac{95}{\mathrm{R_e^{0.983}}} - \frac{96.82}{\mathrm{R_e}}\right)$
21	Romeo (Romeo *et al.* 2002)	2002	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.7065d} - \frac{5.0272}{\mathrm{R_e}}\log\left(\frac{\varepsilon}{3.827d} - \frac{4.567}{\mathrm{R_e}}\log\left({\frac{\varepsilon}{7.7918d}}^{0.9924} + \left(\frac{5.3326}{208.815 + \mathrm{R_e}}\right)^{0.9345}\right)\right)\right)$
22	Sonnad (Olivares *et al.* 2019)	2006	$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\frac{0.4587\mathrm{R_e}}{G^{G/(G+1)}}$ where $G = 0.124\mathrm{R_e}\varepsilon/d + \ln(0.4587\mathrm{R_e})$
23	Rao and Kumar (Brkić 2011b)	2007	$\frac{1}{\sqrt{\lambda}} = 2\log\left[\frac{\left(2\frac{\varepsilon}{d}\right)^{-1}}{\frac{0.444 + 0.135\mathrm{R_e}}{\mathrm{R_e}}\Phi(\mathrm{R_e})}\right]$ where $\Phi(\mathrm{R_e}) = 1 - 0.55e^{-0.33\left(\ln\frac{\mathrm{R_e}}{6.5}\right)^2}$

(*continued*)

Table 2 | continued

No.	Author and source	Year	Equation for rough pipe
24	Buzzelli (Olivares *et al.* 2019)	2008	$\frac{1}{\sqrt{\lambda}}=\frac{B_1+2\log\frac{B_2}{R_e}}{1+\frac{2.18}{B_2}}$ where $B_1=\frac{0.774\ \ln R_e-1.41}{1+1.32\sqrt{\varepsilon/d}}$ $B_2=\frac{\varepsilon}{3.7d}R_e+2.51B_1$
25	Vatankhah and Kouchakzadeh (Brkić 2011b)	2008	$\frac{1}{\sqrt{\lambda}}=0.8686\ln\frac{0.4587R_e}{(G-0.31)^{G/(G+0.9633)}}$ where $G=0.124R_e\varepsilon/D+\ln(0.4587R_e)$
26	Avci (Brkić 2011b)	2009	$\lambda=\frac{6.4}{\left\{\ln R_e-\ln\left[1+0.01R_e\varepsilon/d\left(1+10\sqrt{\varepsilon/d}\right)\right]\right\}^{2.4}}$
27	Papaevangelo (Olivares *et al.* 2019)	2010	$\lambda=\frac{0.2479-0.0000947(7-\log R_e)^4}{\left[\log\left(\frac{\varepsilon}{3.615d}+\frac{7.366}{R_e^{0.9142}}\right)\right]^2}$
28	Brkić (Brkić 2011b)	2011	$\frac{1}{\sqrt{\lambda}}=-2\log\left(10^{-0.4343\beta}+\frac{\varepsilon}{3.7d}\right)$ where $\beta=\ln\frac{R_e}{1.816\ln\frac{1.1R_e}{\ln(1+1.1R_e)}}$
29	Brkić II (Brkić 2011b)	2011	$\frac{1}{\sqrt{\lambda}}=-2\log\left(\frac{2.18\beta}{R_e}+\frac{\varepsilon}{3.7d}\right)$ where $\beta=\ln\frac{R_e}{1.816\ln\frac{1.1R_e}{\ln(1+1.1R_e)}}$
30	Fang *et al.* (Fang *et al.* 2011)	2011	$\lambda=1.613\left[\ln\left(0.234\left(\frac{\varepsilon}{d}\right)^{1.1007}-\frac{60.525}{R_e^{1.1105}}+\frac{56.291}{R_e^{1.0712}}\right)\right]^{-2}$
31	Ghanbari (Asker *et al.* 2014)	2011	$\lambda=\left\{-1.52\log\left[\left(\frac{2.731}{R_e}\right)^{0.9152}+\left(\frac{\varepsilon}{7.21d}\right)^{1.042}\right]\right\}^{-2.169}$
32	Samadianfard (Samadianfard 2012)	2012	$\lambda=\frac{R_e^{\varepsilon/D}-0.6315093}{R_e^{1/3}+R_e\varepsilon/d}+0.0275308\left(\frac{6.929841}{R_e}+\frac{\varepsilon}{d}\right)^{1/9}+\frac{10^{\varepsilon/D}}{\varepsilon/d+4.781616}\left(\sqrt{\frac{\varepsilon}{d}}+\frac{9.99701}{R_e}\right)$
33	Winning and Coole (Winning & Coole 2015)	2014	$\frac{1}{\sqrt{\lambda}}=1.8\log\left[\frac{a_1}{a_2+R_e}+\left(\frac{\varepsilon}{3.73d}\right)^{1.109}\right]^*$

(*continued*)

Table 2 | continued

No.	Author and source	Year	Equation for rough pipe
34	Heydari *et al.* (Heydari *et al.* 2015)	2015	$\frac{1}{\sqrt{\lambda}} = \begin{cases} 0.74 - 0.44\log \mathrm{R_e} - 2.25\log\left(\frac{\varepsilon}{4d}+\frac{6}{\mathrm{R_e^{0.9}}}\right) + 0.053(\log \mathrm{R_e})^2 + 0.0057\left(\log\left(\frac{\varepsilon}{4d}+\frac{6}{\mathrm{R_e^{0.9}}}\right)\right)^2 + 0.046\log\left(\frac{\varepsilon}{4d}+\frac{6}{\mathrm{R_e^{0.9}}}\right)\log \mathrm{R_e} \\ (\mathrm{R_e} < \mathrm{R_{eBL}}) \\ 0.7503 - 1.59\log\frac{\varepsilon}{d} - 0.306\log\left(\frac{\varepsilon}{4d}+\frac{6}{\mathrm{R_e^{0.9}}}\right) + 0.41\left(\log\frac{\varepsilon}{d}\right)^2 + 0.57\left(\log\left(\frac{\varepsilon}{4d}+\frac{6}{\mathrm{R_e^{0.9}}}\right)\right)^2 - 0.98\log\left(\frac{\varepsilon}{4d}+\frac{6}{\mathrm{R_e^{0.9}}}\right)\log\frac{\varepsilon}{d} \\ (\mathrm{R_e} > \mathrm{R_{eBL}}) \end{cases}$ where $\log \mathrm{R_{eBL}} = 2.61 - 1.13\log(\varepsilon/d) - 0.0384[\log(\varepsilon/d)]^2$
35	Mikata and Walczak (Vatankhah 2018)	2015	$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\frac{0.4587\mathrm{R_e}}{G - \ln(G - \ln(G))}$ where $G = 0.124\mathrm{R_e}\varepsilon/d + \ln(0.4587\mathrm{R_e})$
36	Shaikh (Shaikh *et al.* 2015; Brkić 2016)	2015	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{2.51\left(1.14 - 2\log\frac{\varepsilon}{d}\right)^{\alpha}}{\mathrm{R_e}} + \frac{\varepsilon}{3.71d}\right]^{*}$
37	Biberg (Biberg 2016)	2016	$\frac{1}{\sqrt{\lambda}} = 0.8686\left\{\ln\frac{\mathrm{R_e}}{2.18} + \left[\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - 1 + \frac{1}{6x^2}\left(3 + \frac{2}{x}\ln x - \frac{9}{x}\right)\ln x\right]\ln x\right\}$ where $x = \ln\frac{\mathrm{R_e}}{2.18} + \frac{\mathrm{R_e}}{8.0666}\frac{\varepsilon}{d}$
38	Offor and Alabi (Offor & Alabi 2016)	2016	$\frac{1}{\sqrt{\lambda}} = -2\log\left\{\frac{\varepsilon}{3.71d} - \frac{1.975}{\mathrm{R_e}}\ln\left[\left(\frac{\varepsilon}{3.93d}\right)^{1.092} + \frac{7.627}{\mathrm{R_e} + 395.9}\right]\right\}$
39	Brkić and Praks (Brkić & Praks 2018)	2018	$\frac{1}{\sqrt{\lambda}} = 0.8686\left[B - C + \frac{1.0119C}{B + A} + \frac{C - 2.3849}{(B + A)^2}\right]$ where $A = \frac{\mathrm{R_e}}{8.0878}\frac{\varepsilon}{d}$ $B = \ln\frac{\mathrm{R_e}}{2.18}$ $C = \ln(B + A)$
40	Vatankhah (Vatankhah 2018)	2018	$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\left[\frac{0.3984\mathrm{R_e}}{(0.8686s_1)^{\frac{s_1}{s_1 + s_2}}}\right]$ where $s_1 = 0.12363\mathrm{R_e}\frac{\varepsilon}{d} + \ln(0.3984\mathrm{R_e})$ $s_2 = 1 + \left[\frac{1 + s_1}{0.5\ln(0.8686s_1)} - \frac{1 + 4s_1}{3(1 + s_1)}\right]^{-1}$

*No. 33 a_1 and a_2 are constants, and their values are determined using $\mathrm{R_e}$; these values can be found in a determination table proposed by Winning & Coole (2015).
No. 36 The recommended value of α is related to $\mathrm{R_e}$ and ε/d, and the details can be found in Brkić (2016). In the calculation performed in the present study, α is regarded as -0.75, for which the applicable range is $10^4 \le \mathrm{R_e} \le 10^8$ and $10^{-6} \le \varepsilon/D \le 0.05$.

Table 3 | Summary of approximation equations for the friction factor for smooth pipes and transformed equations for rough pipes

No.	Author and source	Year	Equation for smooth pipe	Equation transformed for rough pipe
1	Blasius (Brkić 2012)	1913	$\lambda = \begin{cases} 0.316/\mathrm{R_e}^{0.25} & \text{for } \mathrm{R_e} < 2e4 \\ 0.184/\mathrm{R_e}^{0.2} & \text{for } 2e4 \le \mathrm{R_e} \le 2e6 \end{cases}$	
2	Konakov (Olivares *et al.* 2019)	1950	$\frac{1}{\sqrt{\lambda}} = 1.8\log\mathrm{R_e} - 1.5$	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{5.6234}{\mathrm{R_e}^{0.9}} + \frac{\varepsilon}{3.71d}\right)$
3	Filonenko (Olivares *et al.* 2019)	1954	$\frac{1}{\sqrt{\lambda}} = 1.82\log\mathrm{R_e} - 1.64$	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{6.6069}{\mathrm{R_e}^{0.91}} + \frac{\varepsilon}{3.71d}\right)$
4	Techo *et al.* (Techo *et al.* 1965)	1965	$\frac{1}{\sqrt{\lambda}} = 0.86859\ln\frac{\mathrm{R_e}}{1.964\ln\mathrm{R_e} - 3.8215}$	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{1.964\ln\mathrm{R_e} - 3.8215}{\mathrm{R_e}} + \frac{\varepsilon}{3.71d}\right)$
5	Danish *et al.* (Brkić 2012)	2011	$\frac{1}{\sqrt{\lambda}} = \frac{1}{2}\left(C_o - \frac{1.73718C_o\ln C_o}{1.73718 + C_o} + \frac{2.62122C_o(\ln C_o)^2}{(1.73718 + C_o)^3} + \frac{3.03568C_o(\ln C_o)^3}{(1.73718 + C_o)^4}\right)$ where $C_o = 4\log\mathrm{R_e} - 0.4$	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{1}{\mathrm{R_e}^A} + \frac{\varepsilon}{3.71D}\right) - 0.2A$ where $A = 1 - \frac{1.73718\ln C_o}{1.73718 + C_o} + \frac{2.62122(\ln C_o)^2}{(1.73718 + C_o)^3} + \frac{3.03568(\ln C_o)^3}{(1.73718 + C_o)^4}$ $C_o = 4\log\mathrm{R_e} - 0.4$
6	Fang *et al.* (Fang *et al.* 2011)	2011	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{150.39}{\mathrm{R_e}^{0.98865}} - \frac{152.66}{\mathrm{R_e}}\right)$	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{150.39}{\mathrm{R_e}^{0.98865}} - \frac{152.66}{\mathrm{R_e}} + \frac{\varepsilon}{3.71d}\right)$
7	Li *et al.* (Li *et al.* 2011)	2011	$\frac{1}{\sqrt{\lambda}} = 2\log\left[\mathrm{R_e}\sqrt{\frac{-0.0015702}{\ln\mathrm{R_e}} + \frac{0.3942031}{(\ln\mathrm{R_e})^2} + \frac{2.5341533}{(\ln\mathrm{R_e})^3}}\right] - 0.198$	$\frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{1.25603}{\mathrm{R_e}\sqrt{\frac{-0.0015702}{\ln\mathrm{R_e}} + \frac{0.3942031}{(\ln\mathrm{R_e})^2} + \frac{2.5341533}{(\ln\mathrm{R_e})^3}}} + \frac{\varepsilon}{3.71d}\right]$
8	Taler (Taler 2016)	2016	$\lambda = (1.2776\log\mathrm{R_e} - 0.406)^{-2.246}$	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{1}{\mathrm{R_e}^{0.6388B}} + \frac{\varepsilon}{3.71d}\right) - 0.406B$ where $B = (1.2776\log\mathrm{R_e} - 0.406)^{0.123}$

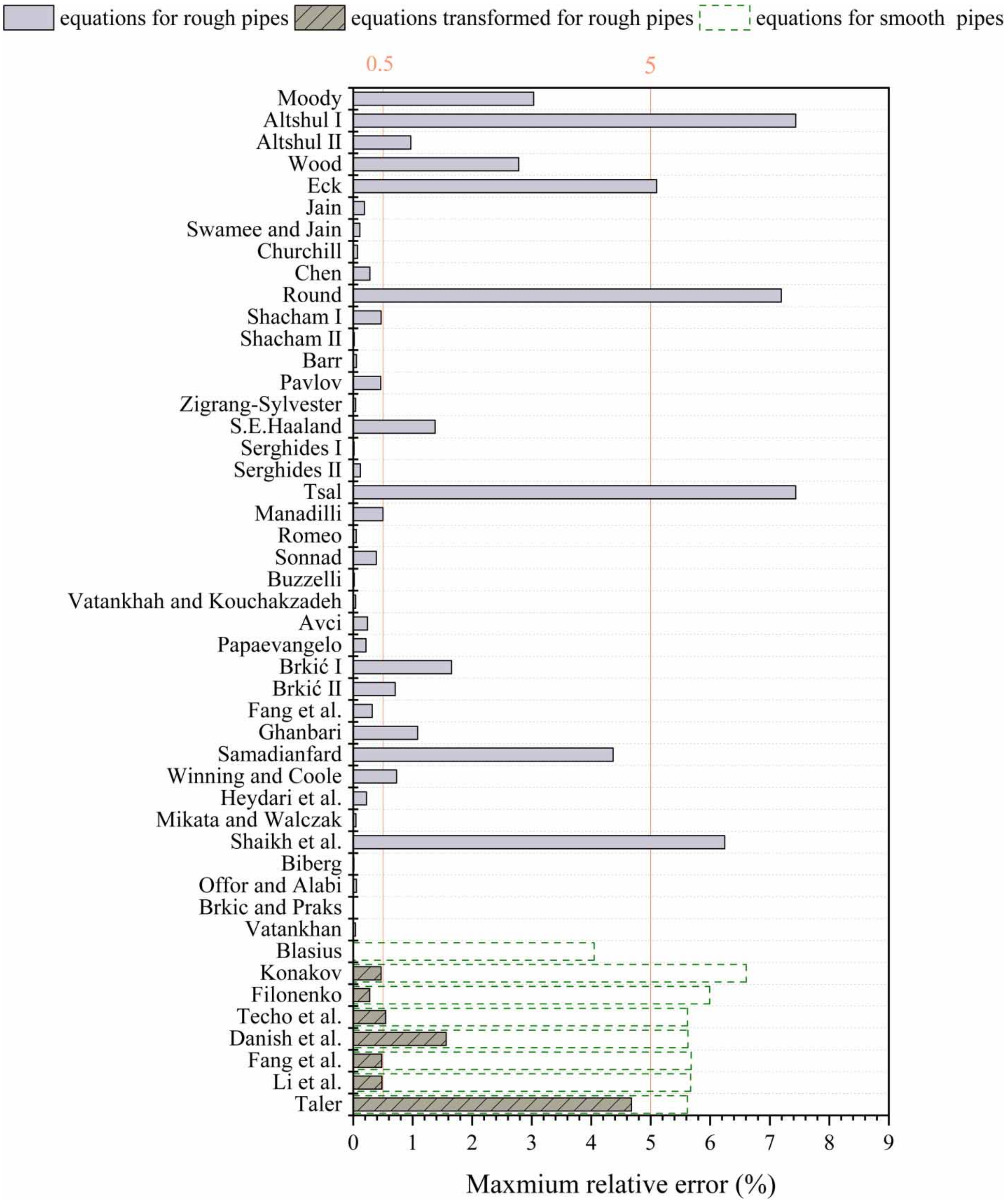

Figure 5 | Maximum relative errors of the approximations. *Note*: The approximation of Rao and Kumar (No. 23 in Table 2) is not shown in this figure; and the approximation of Blasius does not have a corresponding transformed equation.

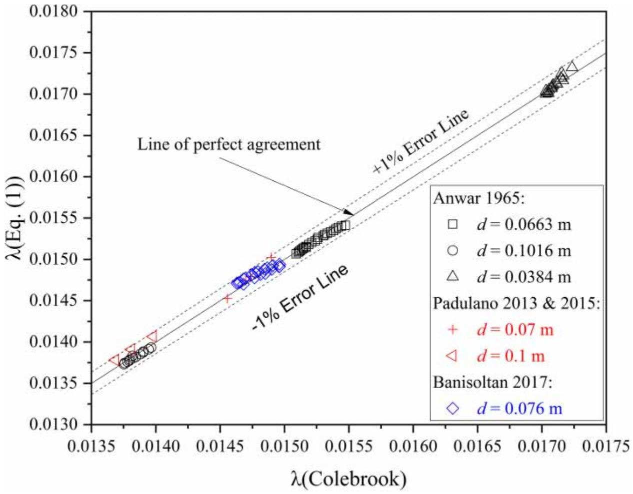

Figure 6 | Comparison between the value of the friction factor calculated using the Colebrook equation and that obtained using Equation (1).

Colebrook equation and Equation (1) is shown in Figure 6. Although the accuracy of Equation (1) is lower than that of certain existing approximations, the error is acceptable.

$$\begin{aligned}\frac{1}{\sqrt{\lambda}} &= 0.608\left(\frac{\varepsilon}{d}\right)^{0.34} + 6.76\left(\frac{h+l}{d}\right)^{-0.34} \\ &+ \left(\frac{\varepsilon}{4.03\mathrm{E}-4\cdot d}\cdot\frac{h+l}{d}\right)^{-3} + 18.35\left(\frac{\varepsilon}{d}\cdot\frac{d}{h+l}\right)^{-0.0409} \\ &+ 30.556\left(\ln\frac{h+l}{d}\right)^{-4.568} - 25.192 \qquad (1)\end{aligned}$$

The partial derivative sensitivity analysis (PDSA), in which a formula is differentiated by input variables, is believed to be capable of assessing the effect of input variables on an equation. According to the approximations for λ of smooth pipes (Table 3), it is clear that λ decreases with an increase in $\mathrm{R_e}$, and according to the transformations for λ of rough pipes (Table 3), λ increases with an increase in ε/d. The form of approximations for λ of rough pipes is more complicated, and thus, a partial derivative analysis must be performed to assess the sensitivity of variables. The partial derivatives of the Brkić and Praks equation (No. 39 in Table 2) were calculated as an example. It was noted that $\mathrm{d}\lambda/\mathrm{dR_e}$ ranges from $-1.76\mathrm{E}{-8}$ to $-3.57\mathrm{E}{-9}$, and $\mathrm{d}\lambda/\mathrm{d}(\varepsilon/d)$ ranges from 6.27 to $1.31\mathrm{E}+1$; i.e., λ increases with a decrease in $\mathrm{R_e}$ or an increase in ε/d. For Equation (1), $\mathrm{d}(1/\sqrt{\lambda})/\mathrm{d}[(h+l)/d]$ ranges from $6.79\mathrm{E}{-2}$ to $2.49\mathrm{E}{-1}$, and $\mathrm{d}(1/\sqrt{\lambda})/\mathrm{d}(\varepsilon/d)$ ranges from $-2.16\mathrm{E}+4$ to $-9.42\mathrm{E}+3$; i.e., λ increases with a decrease in $(h+l)/d$ or an increase in ε/d. This trend is similar to that of the existing approximations.

DISCUSSION

For the data from Humphreys *et al.* (1970) and Zhang (2017), which correspond to the full flow regime in a vertical pipe with a joint barrel, the feasibility of calculating λ of the

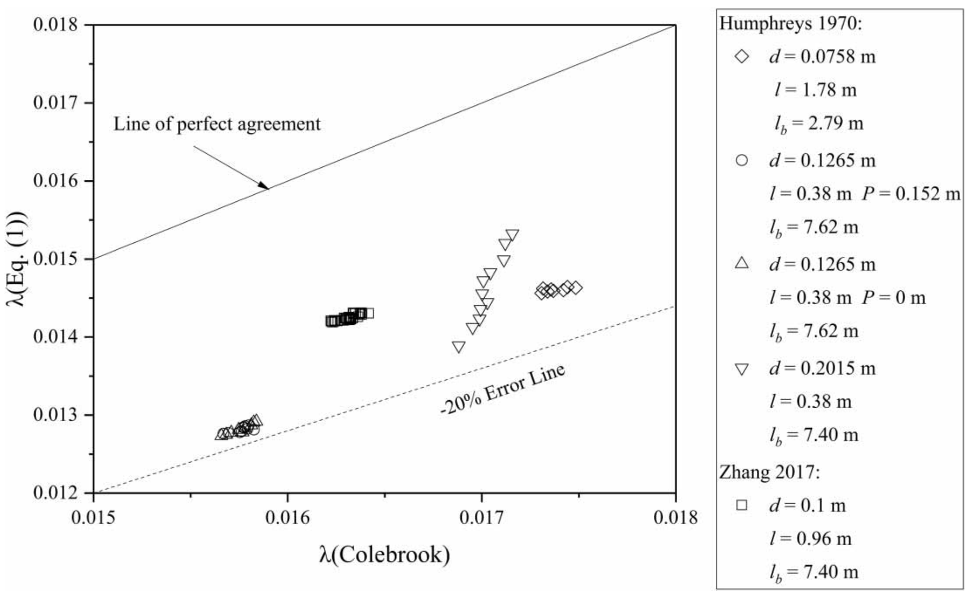

Figure 7 | Comparison between the value of the friction factor calculated using the Colebrook equation and that obtained using Equation (1).

vertical pipe with Equation (1) was verified, and the result is shown in Figure 7. Owing to the existence of the head loss in the joint section (transition loss coefficient) and friction loss in the barrel, the velocity in this case is less than that in a vertical pipe without a joint barrel; thus, the λ calculated using Equation (1) is less than that calculated using the Colebrook equation, as shown in Figure 7. The value calculated using Equation (1) is within the range of $\pm 20\%$ of the value calculated using the Colebrook equation; therefore, it is acceptable to approximate λ using Equation (1) in the full flow regime of the vertical pipe with a joint barrel, when the discharge is unknown and the length of the barrel is not excessively large.

CONCLUSION

We investigated the Darcy–Weisbach friction factor of a full flow regime in vertical pipes based on data from the literature. The existing approximations of λ for a rough pipe were reviewed, and their performance considering the collected data was assessed and classified. The existing approximate equations of λ for a smooth pipe were reviewed and transformed to approximations for a rough pipe; the transformed approximations were assessed, and the results were found to be satisfactory. Furthermore, a new approximation not including the variable R_e was proposed and shown to be very accurate.

ACKNOWLEDGEMENTS

This work was supported by the Natural Science Foundation of Shaanxi Province (2017JZ013); the Leadership Talent Project of Shaanxi Province High-Level Talents Special Support Program in Science and Technology Innovation (2017); the National Key Research and Development Program of China (2016YFC0402404); the National Natural Science Foundation of China (51679197, 41330858 and 51679193).

REFERENCES

Alazba, A. A., Mattar, M. A., ElNesr, M. N. & Amin, M. T. 2012 Field assessment of friction head loss and friction correction factor equations. *Journal of Irrigation and Drainage Engineering* **138** (2), 166–176.

Anwar, H. O. 1965 Coefficients of discharge for gravity flow into vertical pipes. *Journal of Hydraulic Research* **3** (1), 1–19.

Asker, M., Turgut, O. E. & Çoban, M. T. 2014 A review of non iterative friction factor correlations for the calculation of pressure drop in pipes. *Bitlis Eren University Journal of Science and Technology* **4** (1), 1–8.

Banisoltan, S., Rajaratnam, N. & Zhu, D. Z. 2015 Experimental study of hydraulics of drill-drop manholes. *Journal of Hydraulic Engineering* **141** (10), 04015021.

Banisoltan, S., Rajaratnam, N. & Zhu, D. Z. 2017 Experimental and theoretical investigation of vertical drains with radial inflow. *Journal of Hydraulic Engineering* **143** (5), 04016103.

Biberg, D. 2016 Fast and accurate approximations for the Colebrook equation. *Journal of Fluids Engineering* **139** (3), 031401.

Brkić, D. 2011a New explicit correlations for turbulent flow friction factor. *Nuclear Engineering and Design* **241** (9), 4055–4059.

Brkić, D. 2011b Review of explicit approximations to the Colebrook relation for flow friction. *Journal of Petroleum Science and Engineering* **77** (1), 34–48.

Brkić, D. 2012 Can pipes be actually really that smooth? *International Journal of Refrigeration* **35** (1), 209–215.

Brkić, D. 2016 A note on explicit approximations to Colebrook's friction factor in rough pipes under highly turbulent cases. *International Journal of Heat and Mass Transfer* **93**, 513–515.

Brkić, D. & Praks, P. 2018 Accurate and efficient explicit approximations of the Colebrook flow friction equation based on the Wright ω-function. *Mathematics* **7** (1), 34.

Fang, X., Xu, Y. & Zhou, Z. 2011 New correlations of single-phase friction factor for turbulent pipe flow and evaluation of existing single-phase friction factor correlations. *Nuclear Engineering and Design* **241** (3), 897–902.

Fanning, J. T. 1877 *A Practical Treatise on Water-Supply Engineering: Relating to the Hydrology, Hydrodynamics, and Practical Construction of Water-Works, in North America. With Numerous Tables and Illustrations*. Van Nostrand, New York, USA.

Haaland, S. E. 1983 Simple and explicit formulas for the friction factor in turbulent pipe flow. *Journal of Fluids Engineering* **105**, 89–90.

Heydari, A., Narimani, E. & Pakniya, F. 2015 Explicit determinations of the Colebrook equation for the flow friction factor by statistical analysis. *Chemical Engineering & Technology* **38** (8), 1387–1396.

Humphreys, H. W., Sigurdsson, G. & Owen, H. J. 1970 *Model Test Results of Circular, Square, and Rectangular Forms of Drop-Inlet Entrance to Closed-Conduit Spillways*. Report of Investigation 65, Illinois State Water Survey, Urbana, IL, USA.

Kalinske, A. A. 1940 *Hydraulics of Vertical Drain and Overflow Pipes*. Iowa Institute of Hydraulics Research, Iowa City, IA, USA.

Leopardi, M. 2014 Experimental study and design aspects of morning-glory spillways. In: *New Developments in Dam Engineering: Proceedings of the 4th International Conference on Dam Engineering* (M. Wieland, Q. Ren & J. S. Y. Tan, eds), CRC Press, Boca Raton, FL, USA, pp. 461–470.

Li, P., Seem, J. E. & Li, Y. 2011 A new explicit equation for accurate friction factor calculation of smooth pipes. *International Journal of Refrigeration* **34** (6), 1535–1541.

Offor, U. H. & Alabi, S. B. 2016 An accurate and computationally efficient explicit friction factor model. *Advances in Chemical Engineering and Science* **06** (03), 237–245.

Olivares, A., Guerra, R., Alfaro, M., Notte-cuello, E. & Puentes, L. 2019 Experimental evaluation of correlations used to calculate friction factor for turbulent flow in cylindrical pipes. *Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería* **35** (1), 15.

Padulano, R. & Del Giudice, G. 2018 Vertical drain and overflow pipes: literature review and new experimental data. *Journal of Irrigation and Drainage Engineering* **144** (6), 04018010.

Padulano, R., Del Giudice, G. & Carravetta, A. 2013 Experimental analysis of a vertical drop shaft. *Water* **5** (3), 1380–1392.

Padulano, R., Del Giudice, G. & Carravetta, A. 2015 Flow regimes in a vertical drop shaft with a sharp-edged intake. *Journal of Applied Water Engineering and Research* **3** (1), 29–34.

Romeo, E., Royo, C. & Monzón, A. 2002 Improved explicit equations for estimation of the friction factor in rough and smooth pipes. *Chemical Engineering Journal* **86** (3), 369–374.

Samadianfard, S. 2012 Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation. *Journal of Petroleum Science and Engineering* **92–93**, 48–55.

Shaikh, M. M., Massan, S.-u-R. & Wagan, A. I. 2015 A new explicit approximation to Colebrook's friction factor in rough pipes under highly turbulent cases. *International Journal of Heat and Mass Transfer* **88**, 538–543.

Taler, D. 2016 Determining velocity and friction factor for turbulent flow in smooth tubes. *International Journal of Thermal Sciences* **105**, 109–122.

Techo, R., Tickner, R. R. & James, R. E. 1965 An accurate equation for the computation of the friction factor for smooth pipes from the Reynolds number. *Journal of Applied Mechanics* **32** (2), 443.

Vatankhah, A. R. 2018 Approximate analytical solutions for the Colebrook equation. *Journal of Hydraulic Engineering* **144** (5), 06018007.

Winning, H. K. & Coole, T. 2015 Improved method of determining friction factor in pipes. *International Journal of Numerical Methods for Heat & Fluid Flow* **25** (4), 941–949.

Zhang, Z. 2017 *Study on the Characteristics of Drainage Structure of Check Dam and Simulation on the Dynamic Process of Dam Break Induced by Its Failure*. Masters thesis, Xi'an University of Technology, Xi'an, China.

Zigrang, D. J. & Sylvester, N. D. 1982 Explicit approximations to the solution of Colebrook's friction factor equation. *The American Institute of Chemical Engineers Journal* **28** (3), 514–515.

Zigrang, D. J. & Sylvester, N. D. 1985 A review of explicit friction factor equations. *Journal of Energy Resources Technology* **107**, 280–283.

First received 19 December 2019; accepted in revised form 13 March 2020. Available online 27 March 2020