Robust model predictive control for a small reverse osmosis desalination unit subject to uncertainty and actuator fault

Seyed Mohamad Kargar and Reza Mehrad

ABSTRACT

Actuator faults are inevitable in small reverse osmosis desalination plants. They may cause energy losses and reduce the quality of the freshwater, which may endanger human life. Model predictive control (MPC) is a model-based approach widely used to control process systems such as reverse osmosis, while considering a set of constraints. In this paper, three methods of predictive model controllers are considered for the control of a multi-input multi-output (MIMO) reverse osmosis desalination system in the presence of noise, model mismatch, and actuator fault. Formulation of enhanced constrained receding horizon predictive control via bounded data uncertainties (CRHPC-BDU) are extended for linear time-invariant MIMO systems. Permeate flow rate and conductivity of the water produced are controlled by a retentate valve and a bypass valve, respectively. The simulation results show the robustness of the suggested approach in the presence of both noise and uncertainties. CRHPC-BDU has a better performance subject to systems with model uncertainty and actuator fault up to a reasonable limit. By increasing the actuator fault up to 34%, the robustness of CRHPC-BDU is further highlighted in permeate conductivity, where the fluctuations of permeate conductivity dampen sooner than in the other two controllers.

Key words | model predictive controller, passive fault tolerant controller, reverse osmosis desalination system

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INTRODUCTION

Utilizing sea water for drinking is a deep-routed wish for nations that suffer from fresh water shortage. The use of large water purification systems based on reverse osmosis (RO) desalination is growing for the purposes of providing drinking water for the food industry, the wine industry, car washes and other industries. Small RO desalination plants are used in operation rooms, the semiconductor industry and domestic applications.

RO desalination control systems have different tasks for each application. The most challenging part is when a multi-input multi-output (MIMO) system needs to be controlled. A system with more than one input and/or more than one output are known as MIMO. Alatiqi *et al.* (1989) proposed a multi loop control system which includes a pressure controller and two pH controllers. Dynamic models of RO plants were studied by Soltanieh & Gill (1981) and Al-Bastaki & Abbas (1999). The application of some advanced control systems for RO plants were studied by Assef *et al.* (1997); McFall *et al.* (2008b); and Hong Phuc *et al.* (2016). For RO plants many control designs have been studied, such as multi loop proportional-integral (PI) controller (Gambier *et al.* 2006), self-regulating proportional integral derivative (PID) based on genetic algorithms (Kim *et al.* 2008) and feed forward-feedback for disturbance rejection

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(McFall *et al.* 2008a). Model predictive control (MPC) is deployed for many RO plant applications. In Robertson *et al.* (1996) and Janghorban Esfahani *et al.* (2016) an MPC controller based on dynamic matrix control (DMC) was compared with a PID controller for RO plants. Some other MPC approaches are DMC design using system identification (Abbas 2006), MPC design for high capacity RO plant confronting membrane deformation (Bartman *et al.* 2009) and MPC design based on mathematical modelling for an RO plant (Ali *et al.* 2010). Other control methods are also used for RO plants where, for instance in Hong Phuc *et al.* (2016), a robust controller is designed based on two-degrees-of-freedom loop-shaping control methodology.

In addition to problems such as slow variation, model mismatch, measurement noise and disturbances, fault is a likely problem in RO systems. Faults decrease the water quality and increase the risk of hazard. The common faults in RO plants are actuator faults, membrane deformation and sensor faults. The most frequent are actuator faults, in the form of decreasing efficiency which stems from constant exposure to salty water. Fault tolerant control (FTC) systems are divided into active and passive FTCs. In active FTCs, the structure or parameters of the controller change with respect to the estimation of fault detection and isolation (FDI) filters. But passive FTCs are fixed robust controllers that do not use the information of FDI filters and can tolerate faults because they are robust. Active fault tolerant control for RO system was studied by McFall *et al.* (2007) and Gambier *et al.* (2010).

In this work, passive FTC is used to tolerate an actuator fault. It is essential to have a control system with a passive FTC, because a fixed controller has modest hardware and software requirements. Furthermore, in comparison to active FTC systems, passive FTC systems are less complex, and they can be designed to be more reliable based on classical reliability theory (Stoustrup & Blondel 2004). Passive FTCs have implemented in MPC frameworks for RO plants. In Lee (2011) three decades of MPC developments were explained, which supported our primary reason for selecting this control approach. Three MPC controllers, including generalized predictive control (GPC) (Clarke *et al.* 1987a, 1987b), constrained receding horizon predictive control (CRHPC) (Clarke & Scattolini 1991; Chow 1996), and enhanced robustness CRHPC via bounded data uncertainty (CRHPC-BDU) (Ramos *et al.* 2009), were considered. These controllers were designed for the small RO plant studied and identified by Gambier *et al.* (2010). GPC is a well-known controller of the MPC class that is simple, effective, and has a low computational cost, but in general, the stability of this controller is not guaranteed. CRHPC is similar to GPC except for output constraints. These constraints lead to guaranteed stability for exact systems. However, Manoso (1999) showed how GPC and CRHPC nominal stability may fail in systems with uncertainties and cause steady state convergence error accuracy. The CRHPC-BDU guarantees stability in the presence of model and measurement uncertainties. In this work, formulation of the CRHPC-BDU method for MIMO systems is was carried out and is presented in the Supplementary Material. The primary aim of this paper is to propose a novel robust model predictive control (RMPC) to be used in the FTC approach. The suggested approach employs the CRHPC-BDU controller, which is the robust and stable variant of GPC. The state-space model was utilized to design the CRHPC-BDU controller.

PLANT DESCRIPTION

Desalination systems based on RO are commonly used. Considering the system scale and the quality of inlet and outlet water, different units may be deployed for different systems. General RO system units include pretreatment, membrane assembly and post-treatment. The pretreatment unit includes flocculation, chlorination, pH value adjustment, and other treatments that are needed to reduce membrane fouling, inhibit further precipitation and growth of microorganisms, remove suspended particles and assure the safety of the process. A high pressure pump is needed for the feed water to overcome osmotic pressure and permeation of pure water through the membrane. A control valve is also needed to discharge brine water. The permeate water flows into the post-treatment unit to adjust the pH value again, add minerals, exchange ions, etc., as necessary. The whole process involves many control loops which vary from case to case. Two important variables of output are permeate flow rate and permeate conductivity, which are affected by pressure pump and retentate valve.

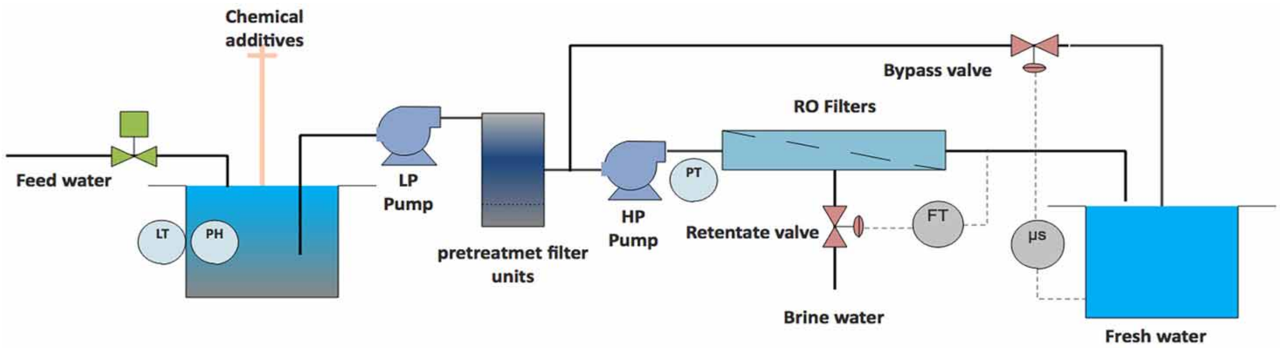

Figure 1 | Schematic of the control loops process.

In this paper a small plant is considered that is used for drinking water purification in which the permeate flow rate is controlled by retentate control valve. The schematic of the case study is presented in Figure 1.

As can be seen in Figure 1, input variables are the brine and bypass flow, which are changed by the retentate and bypass valves, respectively. The input variables control the permeate flow rate and conductivity, which are the output of the system. In order to control permeate conductivity, Gambier *et al.* (2010) suggested that a bypass valve is installed to add a small amount of inlet water to outlet water.

A dynamic linear time invariant (LTI) state space model is identified for plants operating at 250 L/h for the outlet, 500 L/h for the inlet, 0.02 L/h for the bypass flow rates and 425 µS/cm for permeate conductivity. These rates occur when the valve opening for both valves is 50%. The discreet time LTI state space model equations are as follows:

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

$$y(k) = Cx(k) + Du(k) \tag{2}$$

where model parameters for plant in the operation point with 0.015 seconds for the sample time were obtained as Table 1. Although the system is considered to be linear, some authors highlight the nonlinear behavior of RO plants due to membrane fouling. In practice, variation in some parameters, such as membrane fouling that cause nonlinearity, is small and can be eliminated by membrane washing once a week. Thus, it is promising that the behavior

Table 1 | Model parameters

A

$$\begin{bmatrix} 0.201 & 0.010 & 0 & 0 & 0 & 0 & 0 & 0.0002 & -0.001 \\ -3.301 & -0.129 & 0 & 0 & 0 & 0 & 0 & 0.001 & 0.001 \\ 0 & 0 & 0.757 & 0 & 0 & 0 & 0 & 0 & 0.113 \\ 0 & 0 & 0 & 0.955 & 0.116 & 0 & 0 & 0.01 & -0.062 \\ 0 & 0 & 0 & -0.546 & 0.573 & 0 & 0 & 0.110 & -0.606 \\ 0 & 0 & 0 & 0 & 0 & 0.859 & 0.056 & 0 & 0.004 \\ 0 & 0 & 0 & 0 & 0 & -1.338 & 0.043 & 0 & 0.024 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.905 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.286 \end{bmatrix}$$

B

$$\begin{bmatrix} -8.02e-5 & -8.42e-5 \\ -0.001 & -0.001 \\ 0 & 0.009 \\ -0.002 & -0.002 \\ -0.041 & -0.041 \\ 0 & 1.7e-4 \\ 0 & 0.002 \\ -0.632 & 0 \\ 0 & 0.057 \end{bmatrix}$$

C

$$\begin{bmatrix} 222.53 & 12.46 & 0.668 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -21.784 & -7.624 & 12094.53 & 3705.56 & 0 & 0 \end{bmatrix}$$

D

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

of the plant remains quite linear during a week when variations are modeled as small time varying parameter uncertainties. These uncertainties have a small range because they will be eliminated by membrane washing by the end of a week. Moreover, actuator fault is a common experience in this plant, and usually occurs in the retentate valve and affects the control. The presented model is shown below:

$$x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)u(k) + f(k) \tag{3}$$

$$y(k) = (C+\Delta C)x(k) + Du(k) + v(k) \tag{4}$$

where $\Delta A = \delta_A A$, $\Delta B = \delta_A B$, $\Delta C = \delta_A B$ are model uncertainties, $f(k)$ is actuator fault and $v(k)$ is white noise. The control goal is to track reference signal in the presence of uncertainties, sensor noise and actuator fault with small steady state error values.

CONTROL STRATEGIES

Simplicity and transparency of design, flexibility of controller structure and optimal performance are the advantages of the GPC method despite the lack of guaranteed stability. CRHPC provides guaranteed stability for nominal systems. The mathematical descriptions of the GPC and CRHPC approach are presented in the Supplementary Material.

CRHPC-BDU is presented in Ramos *et al.* (2009) for single-input single-output (SISO) systems. In this paper this method is expanded for MIMO systems. The CRHPC-BDU extension guarantees stability in the presence of bounded uncertainties. First, the augmented matrices for the MIMO system are defined. Consider the controlled auto-regressive integrated moving average (CARIMA) representation of a MIMO plant.

$$(A(z^{-1})\Delta)y(z^{-1}) = B(z^{-1})(\Delta u(z^{-1})) + T(z^{-1})n(z^{-1}) \tag{5}$$

where $y\epsilon\mathcal{R}^p$ is output, $u\epsilon\mathcal{R}^m$ is input, $n\epsilon\mathcal{R}^k$ is noise (in a case where $T(z^{-1}) = -A(z^{-1})\Delta$ is measurement noise) and $\Delta = 1 - z^{-1}$. Consider following parameters:

$$g_i^k = \begin{bmatrix} g_{i1}^k & g_{i2}^k & \dots & g_{ip}^k \end{bmatrix}^T \tag{6}$$

$$G_i = \begin{bmatrix} g_i^1 & 0 & \cdots & 0 \\ g_i^2 & g_i^1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_i^N & g_i^{N-1} & \cdots & g_i^1 \end{bmatrix},\ G = \begin{bmatrix} G_1^T & G_2^T & \dots & G_p^T \end{bmatrix}^T \tag{7}$$

$$\bar{G}_i = \begin{bmatrix} g_i^{N+1} & g_i^N & \cdots & g_i^{N-N_u+2} \\ g_i^{N+2} & g_i^{N+1} & \cdots & g_i^{N-N_u+3} \\ \vdots & \vdots & \ddots & \vdots \\ g_i^{N+m_i} & g_i^{N+m_i-1} & \cdots & g_i^{N-N_u+m_i+1} \end{bmatrix},\quad \bar{G} = \begin{bmatrix} \bar{G}_1^T & \bar{G}_2^T & \dots & \bar{G}_p^T \end{bmatrix}^T \tag{8}$$

$$\boldsymbol{w}_i = [\, w_i(t+1) \quad w_i(t+2) \quad \dots \quad w_i(t+N)\,]^T,\quad w = \begin{bmatrix} \boldsymbol{w}_1^T & \boldsymbol{w}_2^T & \dots & \boldsymbol{w}_p^T \end{bmatrix}^T \tag{9}$$

$$\bar{\boldsymbol{w}}_i = [\, w_i(t+N+1) \quad w_i(t+N+2) \quad \dots \quad w_i(t+N+m_i)\,]^T,\quad \bar{w} = \begin{bmatrix} \bar{\boldsymbol{w}}_1^T & \bar{\boldsymbol{w}}_2^T & \dots & \bar{\boldsymbol{w}}_p^T \end{bmatrix}^T \tag{10}$$

$$\boldsymbol{f}_i = [\, f_i(t+1) \quad f_i(t+2) \quad \dots \quad f_i(t+N)\,]^T,\quad f = \begin{bmatrix} \boldsymbol{f}_1^T & \boldsymbol{f}_2^T & \dots & \boldsymbol{f}_p^T \end{bmatrix}^T \tag{11}$$

$$\bar{\boldsymbol{f}}_i = [\, f_i(t+N+1) \quad f_i(t+N+2) \quad \dots \quad f_i(t+N+m_i)\,]^T,\quad \bar{f} = \begin{bmatrix} \bar{\boldsymbol{f}}_1^T & \bar{\boldsymbol{f}}_2^T & \dots & \bar{\boldsymbol{f}}_p^T \end{bmatrix}^T \tag{12}$$

$$\Delta\boldsymbol{u} = \begin{bmatrix} \Delta u(t)^T & \Delta u(t+1)^T & \dots & \Delta u(t+N)^T \end{bmatrix}^T \tag{13}$$

where g_{ij}^k is the k-th step response coefficient of the transfer function linking the j-th input to i-th output, N is prediction horizon, N_u is control horizon, m_i is the number of output constraints of i-th output, $w_i(t+k)$ is the reference signal of i-th output at $t+k$ and $f_i(t+k)$ is the free response of i-th output at $t+k$.

The solution of the optimization problem with the following cost function and constraints is used for

CRHPC-BDU:

$$\min_{\Delta u} \max_{\|\delta G\|<\eta_G} [\| (G+\delta G)\Delta \boldsymbol{u} - \underset{\|\delta e\|<\eta_e}{(e+\delta e)} \|^2 + \rho \| \Delta \boldsymbol{u} \|^2] \quad (14)$$

$$subject\,to: \max_{\substack{\|\delta\bar{G}\|<\eta_{\bar{G}} \\ \|\delta\bar{e}\|<\eta_{\bar{e}}}} \| (\bar{G}+\delta\bar{G})\Delta \boldsymbol{u} - (\bar{e}+\delta\bar{e}) \| = 0 \quad (15)$$

where $e = w - f$, $\bar{e} = \bar{w} - \bar{f}$ and the operators δ and $\| . \|$ define deviation and 2-norm respectively. The input variations $\Delta \boldsymbol{u}$ are calculated using the following equation:

$$\Delta \boldsymbol{u} = \psi G^T e + (I - \psi(G^T G + \lambda_2 I))\Delta u_p \quad (16)$$

In the above equation, Δu_p is the solution particular to the constraints. It is calculated by solving the following nonlinear equations concurrently using a numerical method:

$$\Delta u_p = \bar{G}^T (\bar{G}\bar{G}^T + +\lambda_{\bar{G}} I)^{-1} \bar{e} \quad (17)$$

$$\lambda_{\bar{G}} = \frac{\eta_{\bar{G}} \| \bar{G}\Delta u_p - \bar{e} \|}{\| \Delta u_p \|} \quad (18)$$

and ψ is calculated by the following equation:

$$\psi = H(H^T (G^T G + \lambda_1 I)H)^{-1} H^T \quad (19)$$

where H is the null space of $\bar{G}$. Also λ_1 and λ_2 are calculated by solving the following nonlinear equations concurrently using a numerical method:

$$\lambda_2 = \frac{\rho \| GH\Delta u_f - (e - G\Delta u_p) \|}{\| GH\Delta u_f - (e - G\Delta u_p) \| + \eta_G (\| H \| \| \Delta u_f \| + \| \Delta u_p \|) + \eta_e} \quad (20)$$

$$\lambda_1 = \lambda_2 + \frac{\eta_G \| H \| \| GH\Delta u_f - (e - G\Delta u_p) \|}{\| GH\Delta u_f - (e - G\Delta u_p) \| + \eta_G} \quad (21)$$

$$\Delta u_f = (H^T (G^T G + \lambda_1 I)H)^{-1} (H^T G^T (e - G\Delta u_p) - \lambda_2 H^T \Delta u_p) \quad (22)$$

SIMULATION RESULTS

In this section, the aforementioned three methods, GPC, CRHPC and CRHPC-BDU, are deployed for the RO plant with the purpose of testing and comparing MPC controllers for the RO plant for optimality and robustness. Parameters of the controllers are shown in Table 2 and further descriptions are included in the Supplementary Material. These parameters were adjusted equally to give a fair comparison between control methods and were chosen in such a way that CRHPC stabilizing conditions were satisfied and control signals were not exceeded. For this plant $N_u \geq 11$, $N + 1 \geq m_i > 11$, and $N \geq N_u + 1$ guarantee CRHPC stability, as stated in the Supplementary Material.

The plant was simulated under seven scenarios, as shown in Table 2. The experiments procedure was designed with the aim of studying the plant behavior under various situations, including noise, model mismatch and actuator fault.

Experiment 1: In the first experiment, noise, fault and uncertainty were not included. Small N and N_u, which satisfy the stabilizing conditions of CRHPC, provided a promising performance. It should be noted that large N and N_u impose more computational burden. The results shown in Figure 2 indicate that CRHPC provided slightly faster convergence compared to GPC and CRHPC-BDU. CRHPC had the best performance because of its precise mathematical structure, and CRHPC-BDU had the worst performance because it is designed to be more robust than optimum.

Experiment 2: In the second experiment, model and plant were equivalent except for output measurement noise. Figure 3 shows that CRHPC was very sensitive to measurement noise and had an unacceptable performance despite the guarantee of stability. CRHPC-BDU was more robust and had an acceptable performance. In this case, GPC was recommended, because it had the least computational burden and was robust to noise.

Experiment 3: In the third experiment, N and N_u were increased, which subsequently increased the computational burden. The results in Figure 4 show that bigger horizons improved the controllers performance and made the behavior of CRHPC similar to GPC. CRHPC-BDU settling time decreased significantly in this case.

Table 2 | Controller parameters

		Description	EXP. 1	EXP. 2	EXP. 3	EXP. 4	EXP. 5	EXP. 6	EXP. 7
Horizon parameters	N	Prediction horizon	27	27	100	100	100	100	100
	N_u	Control horizon	25	25	80	80	80	80	80
	m_i	Output condition horizon	12; i = 1,2	12; i = 1,2	12; i = 1,2	12; i = 1,2	12; i = 1,2	12; i = 1,2	12; i = 1,2
Cost function weightings	M_i	Output weighting matrix	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2
	Λ_i	Input weighting matrix in CRHPC	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2	$I_{2\times2}$; i = 1,2
	λ	Input weighting parameter in GPC	1	1	1	1	1	1	1
	ρ	Input weighting parameter in CRHPC-BDU	1	1	1	1	1	1	1
CRHPC-BDU uncertainty bounds	η_G	Maximum deviation of G in CRHPC-BDU	1	1	1	1	1	1	1
	$\eta_{\bar{G}}$	Maximum deviation of $\bar{G}$ in CRHPC-BDU	1	1	1	1	1	1	1
	η_e	Maximum deviation of e in CRHPC-BDU	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	$\eta_{\bar{e}}$	Maximum deviation of $\bar{e}$ in CRHPC-BDU	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Reference signals	w_1	Flow rate reference signal	350 L/h	350 L/h	350 L/h	350 L/h	350 L/h	350 L/h	350 L/h
	w_2	Permeate conductivity reference signal	420 μS/cm	420 μS/cm	420 μS/cm	420 μS/cm	420 μS/cm	420 μS/cm	420 μS/cm
Simulation plant parameters	σ_n	Noise power	0	0.05	0.05	0.05	0.05	0.05	0.05
	f	Actuator fault	0	0	0	0	0	30%	34%
	δ_A	Matched uncertainty in A matrix	0	0	0	0.03	0.04	0.03	0.03
	δ_B	Matched uncertainty in B matrix	0	0	0	0.02	0.03	0.02	0.02
	δ_c	Matched uncertainty in C matrix	0	0	0	0.06	0.07	0.06	0.06

Experiment 4: In the fourth experiment, the model was considered to be uncertain and no fault occurred. The stability and steady state performance of the system could have been be affected by uncertainty and noise. Simulation results in Figure 5 shown that CRHPC-BDU was more robust, with lower fluctuation compared to the others, despite its steady state error.

Experiment 5: In this experiment the amount of uncertainty increased and outputs in Figure 6 show that CRHPC and GPC become unstable while CRHPC-BDU successfully controlled the system.

The five experiments above show that control objectives are reachable for both certain and uncertain conditions. However, for large amounts of uncertainty the performance of controller was not acceptable. In order to study the performance of the design in the presence of described actuator fault, two more experiments were performed for 30 and 34% of valve decay.

Experiment 6: In the sixth experiment, the actuator fault was considered to be 30% at t = 9 seconds. In Figure 7, the effect of the fault is clear, particularly in permeate conductivity. This case shows that controllers had a robust performance for this amount of fault and the quality of responses in CRHPC-BDU was more promising despite the fact that the existence of the fault caused steady state error in responses.

Experiment 7: By increasing the actuator fault up to 34% at t = 9 seconds, the robustness of CRHPC-BDU was further highlighted in permeate conductivity, as shown in Figure 8, where the fluctuations of permeate conductivity dampened sooner in comparison to the other two controlling approaches discussed earlier.

All simulations were performed by a PC with one quad-core Intel Core i7 processor and 16 GB RAM. Computational time values for $N = 100$, $N_u = 80$ and $m = 12$ for

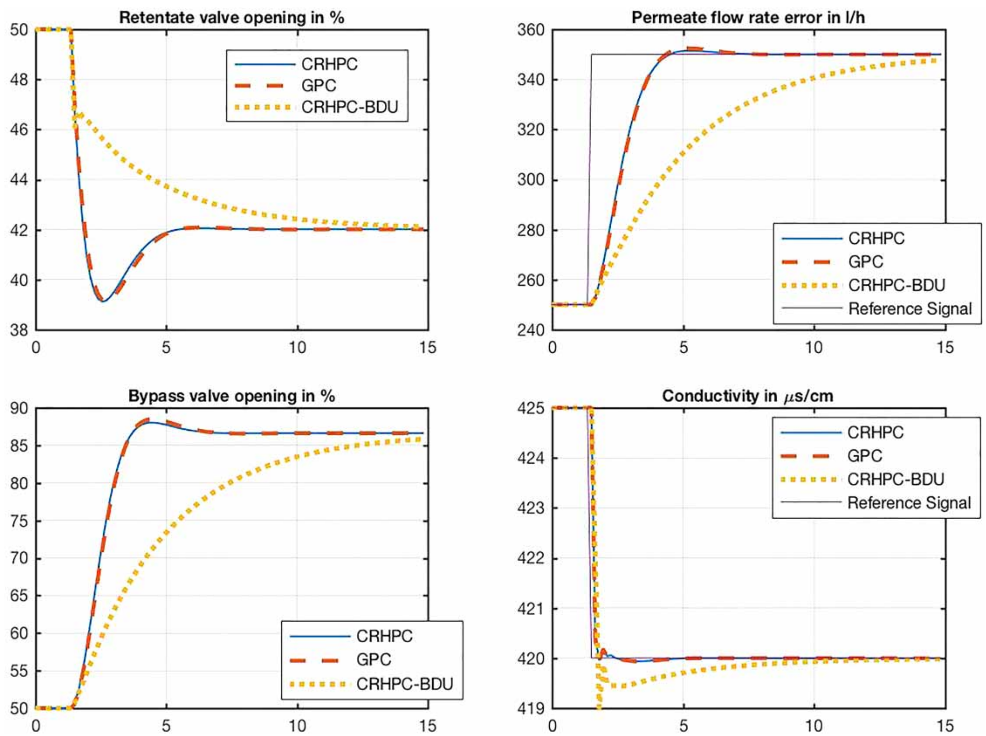

Figure 2 | Input and outputs of the nominal model by three controllers with small computational cost for 15 seconds.

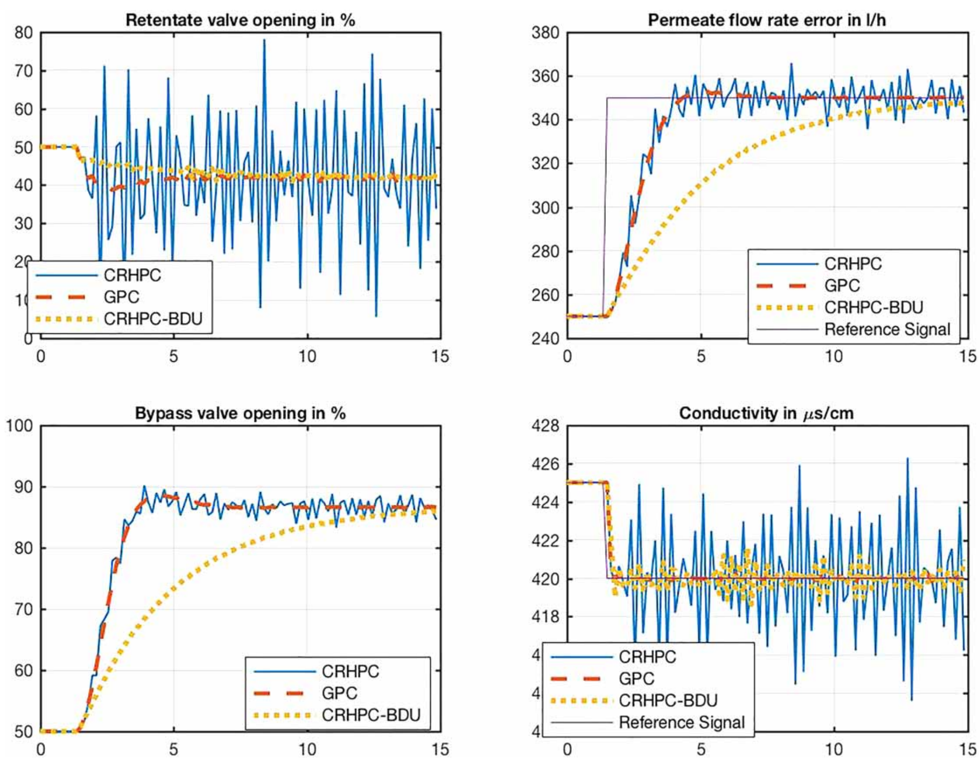

Figure 3 | Input and outputs of plant subject to measurement noise by three controllers with small computational cost for 15 seconds.

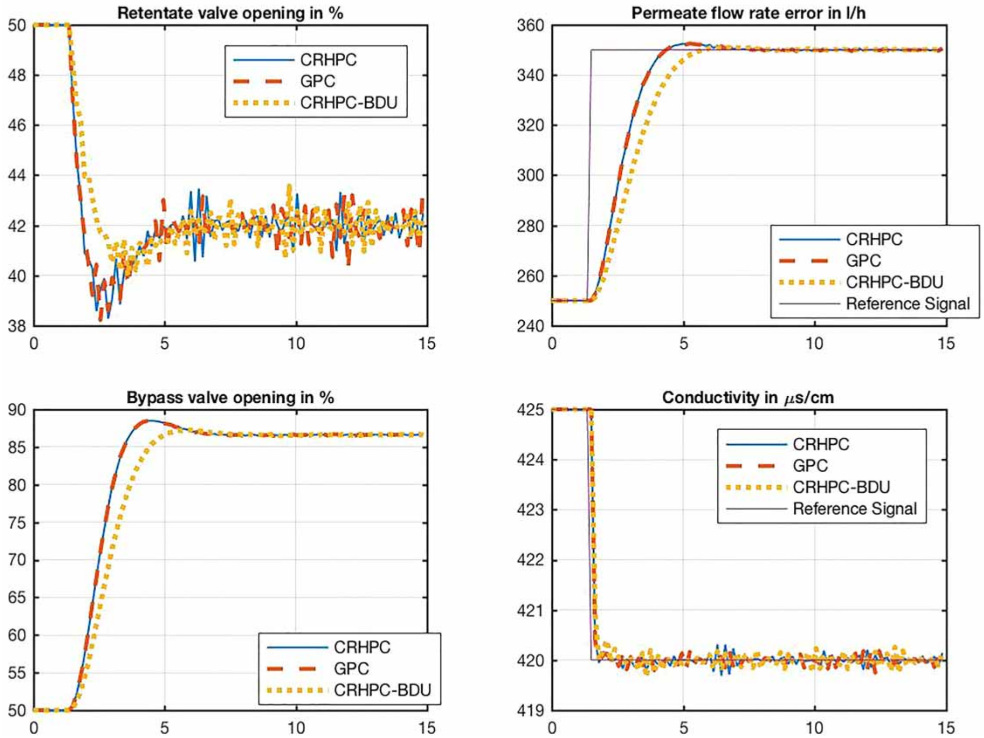

Figure 4 | Input and outputs of plant subject to measurement noise by controllers with increased computational cost for 15 seconds.

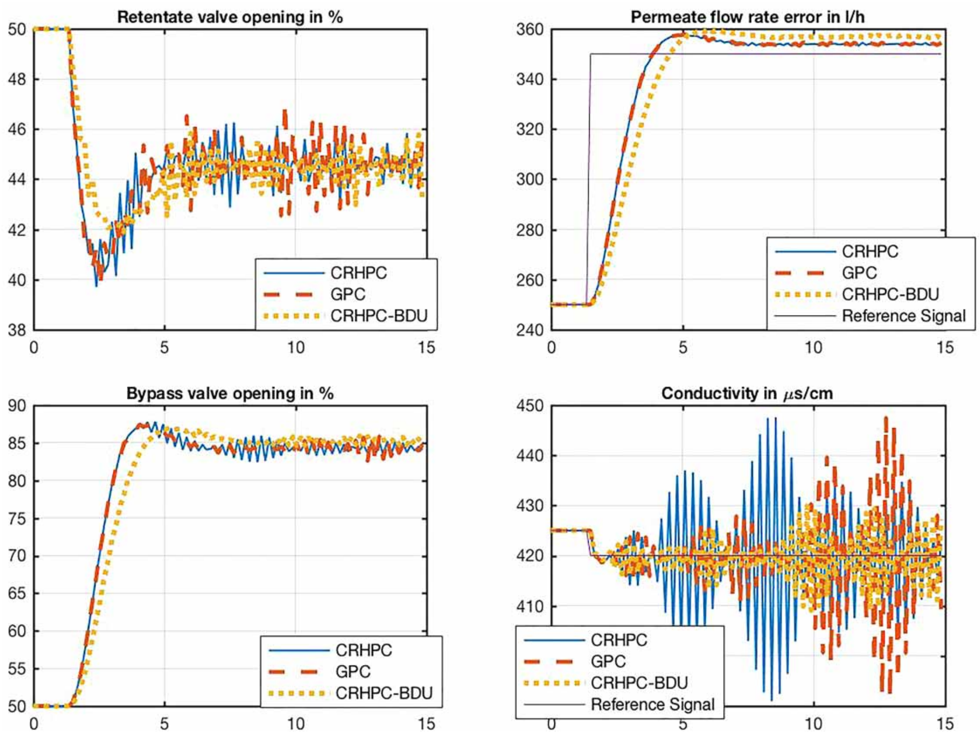

Figure 5 | Input and outputs of plant subject to measurement noise and small uncertainties by three controllers for 15 seconds.

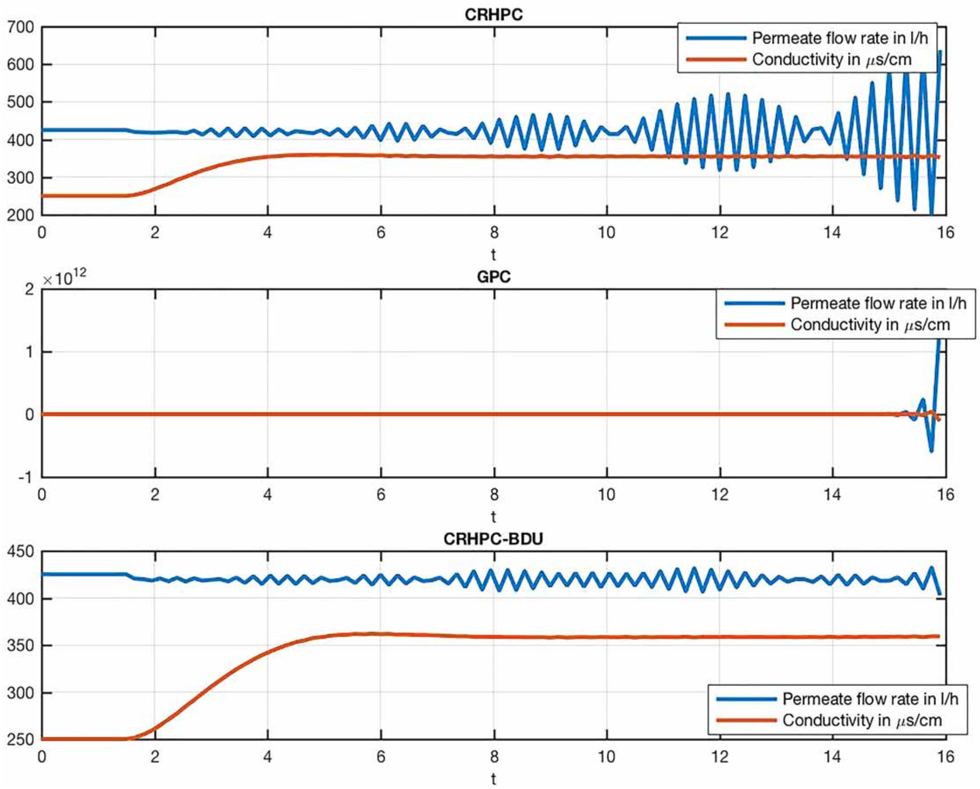

Figure 6 | Outputs of plant subject to measurement noise and large uncertainties by three controllers for 15 seconds.

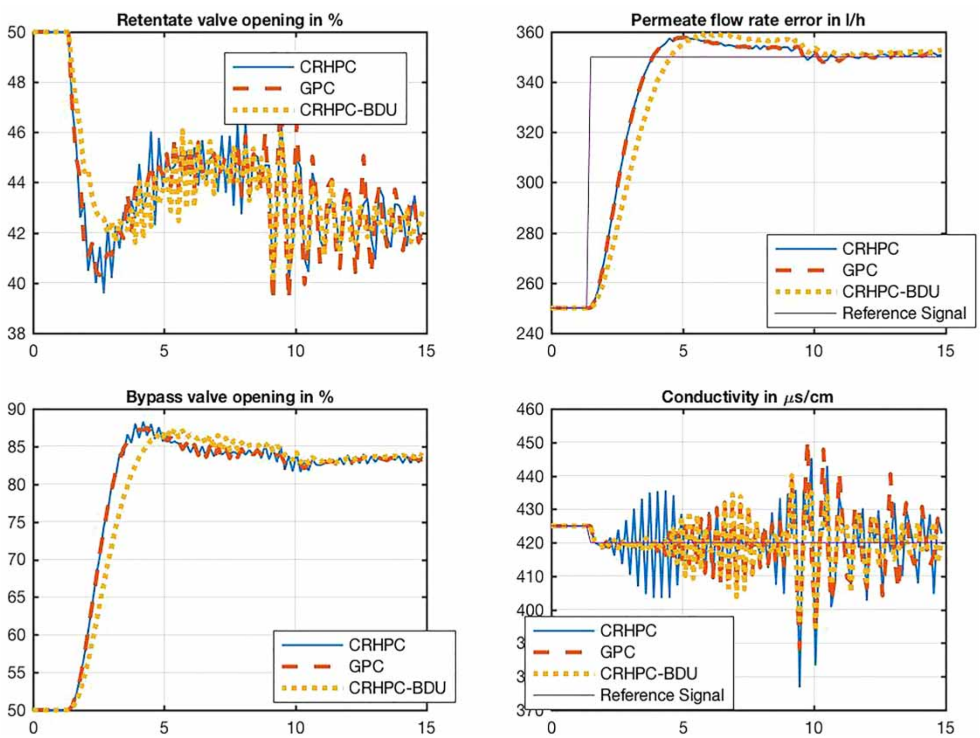

Figure 7 | Input and outputs of plant subject to measurement noise, uncertainties and 30% fault by three controllers for 15 seconds.

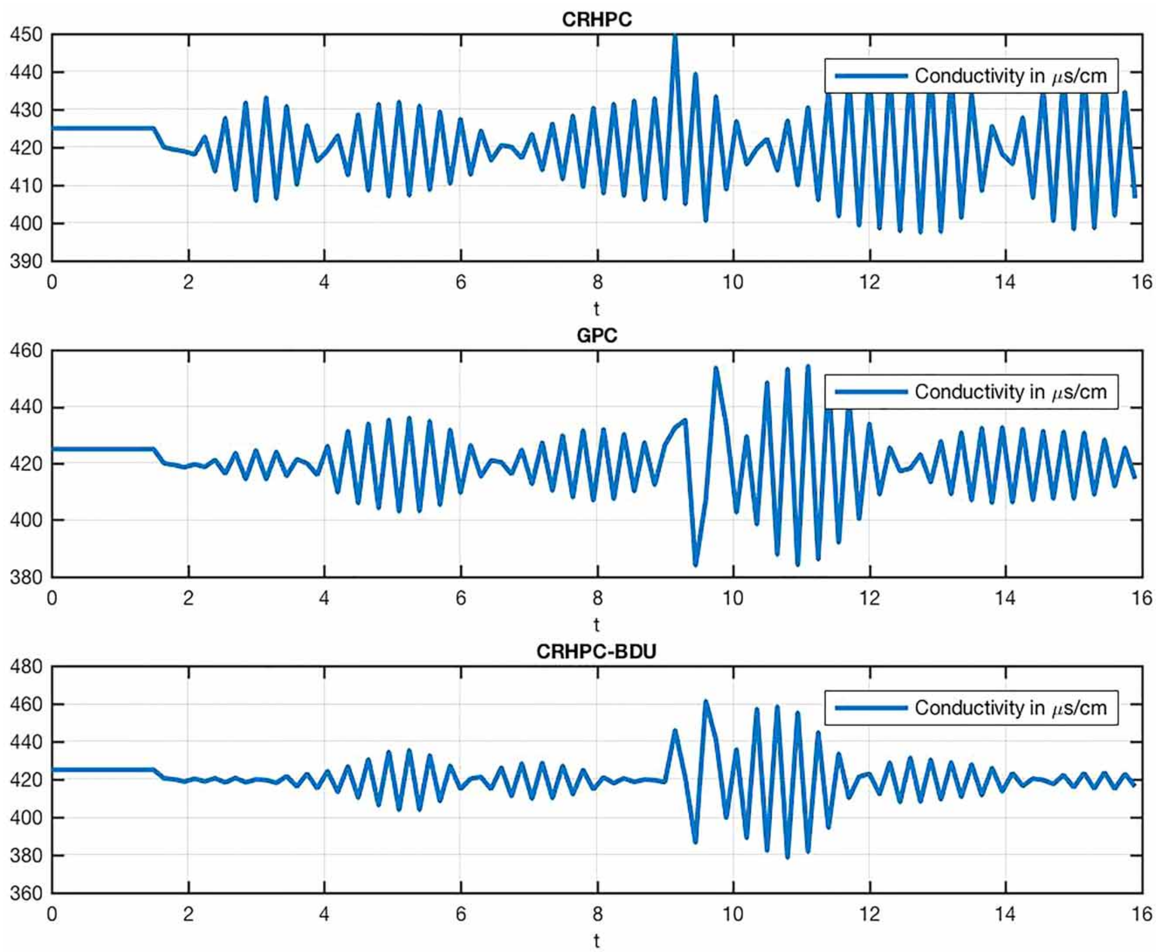

Figure 8 | Permeate conductivity in the context of measurement noise, uncertainties and 34% fault by three controllers for 16 seconds.

each sample of input signals were 0.0081 seconds for GPC, 0.0162 seconds for CRHPC and 0.0812–0.0302 seconds for CRHPC-BDU. Although the time for CRHPC BDU is greater than for the other two methods because of numerical optimization loops, it is acceptable in comparison to the plant sample time (0.15 seconds). Further experiments showed that for faults larger than 34% all controllers were not able to dampen the fluctuations.

CONCLUSION

In this paper, the comparative study between the three controllers, CRHPC, GPC, and CRHPC-BDU, were studied to show the performance of the proposed approach. These controllers were analyzed by their closed-loop stability, nominal performance, measurement noise, model mismatch, and capability of being used as passive FTCs for small actuator faults. Moreover, an expanded version of CRHPC-BDU for MIMO systems was proposed. The simulation was performed to show that the proposed method can tolerate the fault because of its robustness. The actuator fault was increased in the simulation model to measure the performance of the experiment.

The results revealed that GPC had the least computational cost with poor performance, and CRHPC-BDU provided robustness for controlling the system, though with more computational cost. CRHPC provided nominal stability, while its performance was not better than the other two methods. These controllers were used as passive FTCs for the RO plant with uncertainty in the model. CRHPC-BDU gave better results than the other two controllers, and the actuator fault was tolerated up to 34%.

NOTES

The authors declare no competing financial interest.

SUPPLEMENTARY MATERIAL

The Supplementary Material for this paper is available online at https://dx.doi.org/10.2166/ws.2020.043.

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