Developing a surface water resources allocation model under risk conditions with a multi-objective optimization approach

Mohammad Taghi Aalami, Vahid Nourani and Hamid Fazaeli

ABSTRACT

One of the major socioeconomic and global sustainability issues is water scarcity, which imperils human survival and regional development. The current study aims to develop a model for allocating water resources more efficiently and equitably. In this regard, a multi-objective programming approach was developed with the first objective of equality of water resource allocation to be maximized, and the second objective of risk to be minimized. The risk considered in this study was the economic efficiency loss risk. For the annual water allocation model, the fluctuation in available water within the river basin is the main source of uncertainty and can result in the corresponding risk of economic efficiency loss. Thus, it is essential to manage the economic efficiency loss risk resulting from uncertainty. To solve the model, the compromise programming (CP) method was used. A sustainability index was also employed to determine the objective function weights. The developed model was applied to the Givi River basin in Iran. From the results, it was found that using the sustainability index is a suitable strategy in the CP method for determining the objective function weights. The results showed that the proposed model can be helpful in water management to allocate water resources.

Key words | Conditional Value-at-Risk (CVaR), CP method, Gini coefficient, multi-objective, water resources allocation

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INTRODUCTION

Population and economic growth has increased agricultural and domestic water demand. Different groups of water users need to know how much water will be allocated to their activities. Consumers also need to know how much water, allocated to them, may not be supplied. They can then buy water from a more expensive source or decrease their development activities. Two important criteria in optimal allocation of water resources are efficiency and equality. Together with these two criteria, consideration should be given to sustainable water allocation (Hu *et al.* 2016a). Numerous researchers have been interested in developing an efficient index to show the equality in distribution of water resources. One of the first pioneers was Gini (1921).

In recent decades, water resource researchers have been attempting to use the Gini coefficient in demonstrating the equality of water distribution. Also, the efficiency as an essential indicator should always be considered alongside the equality index. If only equality is taken into account, regardless of efficiency, it will not lead to economic development. Along with these factors, the issue of sustainability must always be considered. In fact, sustainability is a guarantee of continued progress. The word sustainability has assumed a variety of meanings. Once the water resource system is simulated using hydrological inputs, the time series values of these system performance criteria can be derived. These time series values can be summarized using the statistical measures of reliability, resilience and

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vulnerability. The relative sustainability of the system with respect to each of these criteria is higher the greater the reliability and resilience, and the smaller the vulnerability (Loucks 1997; Ioslovich & Gutman 2001; Garcia & Pargament 2015; Ha & Gao 2017). Given that water allocation issues are always uncertain, it is inevitable to consider uncertainties that lead to risk conditions. The most important uncertainty of water resources allocation is the volume of available resources. The variation in the volume of water supply is always a source of uncertainty. The next uncertainty is the level of water demand in different sectors (i.e., agricultural, domestic, and industrial sectors). The occurrence of uncertainty in different sectors occurs for different reasons. For example, the demand of the agricultural sector is variable due to changes in the area and pattern of cultivation (Qian *et al.* 2014). Three risk criteria for evaluating the possible performance of water resources systems are reliability, resiliency, and vulnerability. Reliability indicates how likely a system is to fail, resiliency implies how quickly it recovers from failure, and vulnerability demonstrates how severe the consequences of failure may be. It is unlikely that a single mathematical definition of these concepts will be appropriate or useful in all situations. These criteria can be used to assist in the assessment and selection of alternative design and operating policies for different water resource projects (Hashimoto *et al.* 1982). Sandoval-Solis *et al.* (2010) presented a water resources sustainability index that makes it possible to assess and compare different water management policies with respect to their sustainability. The sustainability index describes policies that maintain or improve the desired water management characteristics of the basin in the future.

Liu *et al.* (2018) formulated water supply uncertainties, and then evaluated risks related to droughts and sudden water pollution. They showed that the water supply problem could be alleviated to some extent by increasing the distance between the pollution location and the reservoir release gate. Qian *et al.* (2014) presented evaluation criteria of risk between water supply and water demand which includes threat, susceptibility, and vulnerability. They developed a model for risk evaluation based on the maximum entropy principle and discriminant analysis. Zeying *et al.* (2015) described uncertainty of water resources systems and quantitative characterization methods of risk analysis, including three criteria (reliability, resiliency, and vulnerability). They finally provided a decision support of risk analysis for researchers, policy-makers and stakeholders of water resources systems. The Conditional Value-at-Risk (CVaR) criterion is used to consider the system under the risk condition. This criterion covers the important part of the existing uncertainties. CVaR is derived from the loss distribution function (Rockafellar & Uryasev 2002). Yamout (2005) compared the different probability and non-probability based analytical techniques used in risk management, focusing on the Conditional Value-at-Risk method. She investigated the impact of incorporating the CVaR on analyzing a water allocation problem versus using the frequently used expected value, two-stage modeling, scenario analysis, and linear optimization tools. She developed five models to examine the water resource allocation when available supplies are uncertain. The inclusion of the CVaR objective function provides for the optimization and control of high-risk events. Minimizing CVaR does not, however, permit control of lower-risk event behavior with respect to the confidence level, when compared to value-at-risk. Hu *et al.* (2016b) developed a multi-objective model involving water allocation equality and economic efficiency risk control to help water managers mitigate water allocation problems. They introduced the Gini coefficient to optimize water allocation equality in water use sectors and CVaR to control the economic efficiency loss risk corresponding to variations in water availability. Due to uncertainties in water supply and water demand, water allocation under the risk condition is unavoidable. In this study, we attempted to focus on the equal and efficient allocation of water under the risk (the economic efficiency loss risk) condition. Generally, high risk is positively correlated with high returns as well as high losses. Therefore, a high fluctuation in available water resources exacerbates the risk of water management system failure. As a result, the economic efficiency loss risk control is an important element for a viable and valid solution. Thus, it is essential to manage the economic efficiency loss risk resulting from uncertainty. In order to model the water allocation issue, multi-objective optimization with two objective functions of CVaR and Gini was used. Also, the compromise programming (CP) approach was used to solve the multi-objective optimization problem.

An important step in the CP method is determining the weight of the objective functions, however this step is barely considered in the existing literature. In this research, for determining the weight of the objective functions, the sustainability index was proposed.

MATERIALS

Study area and data source

The Givi River is one of the branches of the Ghezel Ozan River, which originates in the Aq Dag mountains. This river is located in the southern part of Ardebil province in Iran. The river enters the Ghezel Ozan River basin after joining the Sangvar River in the lower reaches, finally it enters the Caspian Sea. The Givi River basin has an area of 600 km^2 (see Figure 1).

The whole study area was divided into the two sub-basins of the Sangvar and Givi Rivers. Considering the length of the observation data period at the Firozabad base hydrometric station, a 50-year period (i.e., 1960–2010) was selected for the study. The study area consists of one urban area and 104 rural areas. The expected value of the annual water supply in that area is approximately 103 million m^3. The minimum requirement for domestic water in rural and urban areas is 2.2 million m^3, and the minimum requirements for the industrial and agricultural sectors are

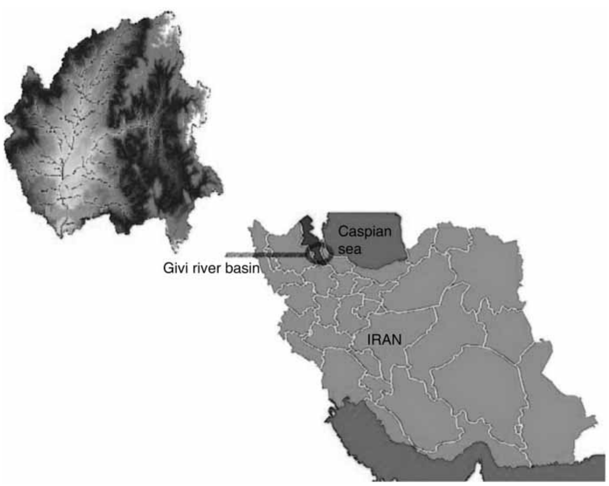

Figure 1 | The Givi River basin in Ardabil province.

40 and 34 million m^3, respectively. Agricultural demand in the Sangvar sub-basin is higher than in the Givi sub-basin. In contrast, the industrial water demand in the Givi basin is higher than in the Sangvar. All the necessary data for this study were taken from the Regional Water Company of Ardabil province.

Methodology

Decision making is an integral part of human lives. It ranges in scope from the individual to the largest groups (Chankong & Haimes 1983). In recent decades, researchers have focused on multi-criteria decision making (MCDM) for complex decisions. These decision models are divided into two major categories: multi-objective models (MODMs) and multi-attribute models (MADMs). Water allocation is a multi-criteria decision-making problem that has always been highly regarded in literature (Cohon & Marks 1973; Hipel 1992; Raju et al. 2000; Babel et al. 2005; Xevi & Khan 2005; Atiquzzaman et al. 2006; Han et al. 2011; Ren et al. 2019). The conceptual framework of a water allocation model is shown in Figure 2.

Compromise programming (CP) technique

There are several methods to solve multi-objective models. In this study, a compromise programming method was used. The compromise programming method was first used by Zeleny (1973). Subsequently, many researchers developed compromise programming for water allocation (Bella et al. 1996; Raju et al. 2000; Shiau & Wu 2006; Zarghaami 2006; Fattahi & Fayyaz 2010; Read et al. 2014; Roozbahani et al. 2015; Salman et al. 2019). The CP method identifies the solutions which are closest to the ideal solution, as determined by some measure of distance. Due to its simplicity, transparency and easy adaptation to both continuous and discrete settings, the CP method is recommended as the multi-objective analysis method of choice for application in water resources systems management (Simonovic 2009). This method is based on scaling of the outcome for each criterion and subsequently calculating a weighted sum of metric distance for each criterion for making a single objective function. The solutions obtained from the CP method have been found to be the closest to

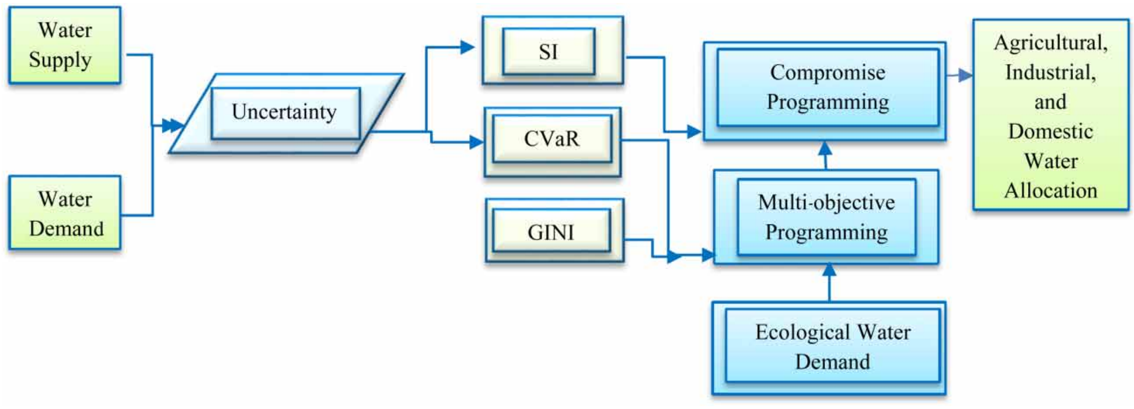

Figure 2 | Conceptual framework of water allocation model.

the ideal (Hu *et al.* 2016c). For a model with n objective functions, the compromise programming distance metric is presented as below:

$$Minimize: L_P = \left[\sum_{j=1}^{m} w_j^P \left(\frac{f_j^* - f_j(\bar{x})}{f_j^* - f_{wj}}\right)^P\right]^{\frac{1}{P}} \tag{1}$$

where the value w_j^P (the weight of objective j) reflects the decision makers' preference for the importance of the objective; $f_j(\bar{x})$ is the calculated value of the objective function j; f_j^* and f_{wj} are the most optimal value and the most inferior value; P (=1, 2, $\cdots$ ∞.) is a metric parameter which reveals the significance of the maximum deviation from the ideal point (Simonovic 2009).

Applying the Gini coefficient to measure inequality

Equal access to water or the benefits of water use will result in sustainable development and elimination of poverty in developing countries (DWAF 2005; Cullis & Van Koppen 2007). An indicator for equal distribution was first introduced by Gini (1921). The Gini coefficient is one of the most commonly used indicators for measuring distribution. It is traditionally applied to the measurement of income inequality, however, it is also applied to measure land inequality. Subsequently, many researchers have used this indicator for water resources allocation equality (Cullis & Van Koppen 2007; Wang *et al.* 2012; Hu *et al.* 2016c; Dai *et al.* 2018; Lee *et al.* 2019). The Gini coefficient is calculated from un-ordered size data (x_i) as the 'relative mean difference', i.e., the mean difference between every possible pair of individuals ($|x_i - x_j|$), divided by the mean size and is defined as follows:

$$Gini = \frac{1}{2n^2\bar{x}} \sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j| \tag{2}$$

The Gini coefficient can be displayed graphically using a Lorenz curve. It was developed by Max Lorenz in 1905 for representing inequality of wealth distribution (Figure 3).

In order to use the Gini coefficient for water allocation, it is necessary to form a matrix, which actually represents the water allocation matrix, in which the entry of that matrix x (i, j) represents the volume of water allocated to the sector i in sub-basin j. Given the existence of two sub-basins and three consumers in this study, the above matrix

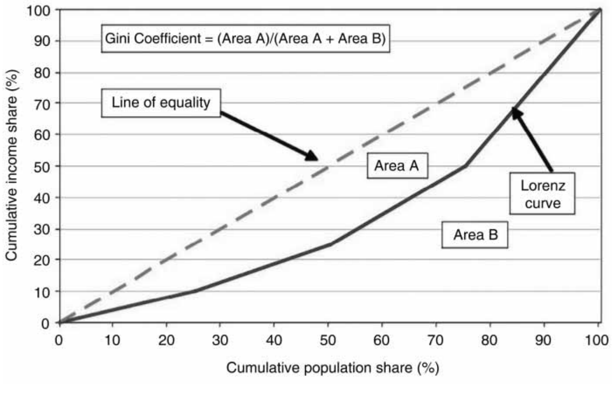

Figure 3 | Lorenz curve and Gini coefficient (Cullis & Van Koppen 2007).

would be a matrix of 3 by 2. The sub-basins are Givi and Sangvar, respectively and the sectors are domestic, industrial, and agricultural, respectively. Formation of the Gini coefficient for the domestic sector (D) is based on the population of the sub-basins (POP), for the agricultural sector (A) it is based on the area under cultivation (s) and for the industrial sector (I) it is based on the average economic return per unit volume of allocated water (B). Based on the above description, the relationship is as follows (Hu *et al.* 2016c):

$$\mathrm{Gini}_{\mathrm{D.A.I}}(\mathrm{x}) = \left| v_1 \frac{\left|\frac{x_{11}}{POP_1} - \frac{x_{12}}{POP_2}\right|}{\left(\frac{x_{11}}{POP_1} + \frac{x_{12}}{POP_2}\right)} + v_2 \frac{\left|\frac{x_{21}}{s_1} - \frac{x_{22}}{s_2}\right|}{\left(\frac{x_{21}}{s_1} + \frac{x_{22}}{s_2}\right)} - v_3 \frac{\left|\frac{x_{31}}{B_{31}} - \frac{x_{32}}{B_{32}}\right|}{\left(\frac{x_{31}}{B_{31}} + \frac{x_{32}}{B_{32}}\right)} \right| \tag{3}$$

where v_1, v_2, v_3 are the weights and $v_1 + v_2 + v_3 = 1$.

Conditional Value-at-Risk as a measure of risk (CVaR)

For calculation of risk the Conditional Value-at-Risk method was used. Value-at-Risk, which is denoted as VaR_α, is defined as the maximum potential loss with a confidence level α and Conditional Value-at-Risk, which is denoted as $CVaR_\alpha$, measures the expected value of losses exceeding VaR_α. CVaR is both coherent and expectation-bounded. It is a simple representation of risk and accounts for risk beyond VaR_α, making it more conservative than VaR_α (Yamout 2005; Rahimi & Ghezavati 2018; Dixit & Tiwari 2020).

The VaR_α of the loss associated with a decision x is defined in the following way: $\xi(x)$ (the VaR value)

$$\xi_\alpha(x) = min\{\xi | \Psi(y, \xi) \geq \alpha\} \tag{4}$$

$$\Psi(y, \xi) = prob\{y | L(y, \omega) < \xi\} \tag{5}$$

where $\mathrm{L}(\mathrm{y}, \omega)$ is the loss function with ω being the stochastic factor, prob denotes probability, and $\Psi(y, \xi)$ represents the cumulative probability distribution function of the loss function. $CVaR_\alpha$ (expected value of losses exceeding (VaR_α) is defined as follows:

$$CVaR(x) = E\{f(x, y) | L(y, \omega) > \xi_\alpha(x)\} \tag{6}$$

where E is the expected mean. From Equation (6), CVaR can be obtained as follows:

$$CVaR = \xi_\alpha + \frac{1}{1-\alpha} \sum\nolimits_{n=1}^{N} P_n(L_n - \xi_\alpha) \tag{7}$$

$$L_n = 1 - \frac{X \odot B}{(\tilde{Q}_n - EWD) * (\max(B))} \tag{8}$$

where X is the water allocation matrix previously discussed, B is the economic return matrix, EWD is ecological water demand, n is the number of probability-value pairs in the discrete approximation, L_n is loss function, P_n is probability of Q_n, and $\odot$ is the Hadamard product of two matrices (Rockafellar & Uryasev 2002). Figure B in the Supplementary Material demonstrates VaR and CVaR deviations.

Water resources system sustainability

According to the WCED (1987), sustainable water resource systems are those designed and managed to contribute fully to the objectives of society, now and in the future, while maintaining their ecological, environmental and hydrological integrity. Sustainability is more a social goal than a scientific concept. It implies an ethic. Public value judgments must be made about which demands and wants should be satisfied today and what changes should be made to ensure a legacy for the future. Different individuals have different points of view, and it is the combined wisdom of everyone's expressed opinions that will shape what society may consider sustainable (Loucks 1997; Gohari *et al.* 2017; Chen *et al.* 2018).

Many researchers were planning to develop an aggregate index for the combination of water resource systems performance (Palmer 1965; Brown *et al.* 1972; Reiquam 1972; Milbrink 1983). Loucks (1997) offers the following method, based on measures of risk and uncertainty:

$$SI = Rel \times Res \times (1 - vul) \tag{9}$$

where SI is a summary index, which measures the sustainability of water resources systems, and Rel^i, Res^i *and* Vul^i

represent reliability, resiliency and vulnerability respectively. In this study Rel^i, Res^i and Vul^i were calculated for each water year (hydrological year). The data used were monthly (monthly available data from the river in cubic metres), and in each month (simulated period) the volume of total available surface water was compared with the total water demand. Reliability indicates a successful period probability, resiliency is the probability that a successful period follows a failure period, and vulnerability demonstrates how significant the likely consequences of failure may be. If the available water meets the water demand it is a successful period otherwise it is a failure period. According to Loucks (1997) definition, reliability, resiliency, and vulnerability can be expressed as follows:

$$\text{Reliability} = (\text{Number of time periods that water supply meets demand/Total number of simulated periods}) \quad (10)$$

$$\text{Resiliency} = (\text{Number of successful periods following a failure period/Number of failure periods}) \quad (11)$$

$$\text{Vulnerability} = (\text{The mean amount of water supply deficit in simulated periods/Water demand}) \quad (12)$$

Model construction

Generally, one or more stochastic parameters exist in the structure of risk-based models. The stochastic parameter which is used in this study is $\tilde{Q}$. This parameter is the total available water from the river. The rest of the parameters are deterministic. The decision variables used in the model are decision vector X. To solve the multi-objective compromise, the weight of the objective functions should be calculated. In this study, the sustainability index (SI) is used to calculate the weights. The frequently used model variables are listed in Table 1. Thus, the objective functions are formulated as follows:

$$OF_1 = Gini_{D.A.I}(x) \quad (13)$$

$$OF_2 = CVaR(x) \quad (14)$$

$$\text{compromise } OF = [SI \times Gini_{D.A.I}(x) + (1 - SI) \times CVaR(x)] \quad (15)$$

In Equation (15), the sustainability index (SI) is the weight of the Gini coefficient and (1-SI) is the weight of CVaR. As the SI increases, the weight of the Gini function increases and the weight of the CVaR function decreases. In other words, with an increasing SI, the system stability increases. In a more stable system, due to the high efficiency level, it is better to pay more attention to the equality criterion. Conversely, with the system stability decreasing to reach the optimum point, more attention should be paid to the CVaR criterion. To formulate the model structure, the constraints should be considered as follows:

Table 1 | Description of the model variables

Variable	Description
i	Index of sector (domestic, agricultural and industrial sectors)
j	Index of sub-basin
D	Index of the domestic sector
A	Index of the agricultural sector
I	Index of the industrial sector
x_{ij}	The amount of water allocated to sector i in sub-basin j
$Gini_{D.A.I}(x)$	The Gini coefficient in domestic, agricultural and industrial sectors
POP_j	Population in sub-basin j
s_j	Area under cultivation in sub-basin j
B_{ij}	Average economic return per unit volume of allocated water of sector i in sub-basin j
$CVaR$	Conditional Value-at-Risk
SI	Sustainability index
Rel	Reliability
Res	Resiliency
Vul	Vulnerability
EWD	The ecological water demand
$\tilde{Q}$	The stochastic parameter that represents the total available water from the river
$minD(i, j)$	The minimum water demand for sector i in sub-basin j
$maxD(i, j)$	The maximum water demand for sector i in sub-basin j

Total allocated water constraint

The total water allocated to agricultural, domestic and industrial sectors, and ecological water demand (EWD) should be less than or equal to the expected value of available water:

$$\sum_{i=1}^{i}\sum_{j=1}^{j} X + EWD \leq E(\tilde{Q}) \tag{16}$$

where $E(\tilde{Q})$ is the expected value of the stochastic parameter Q.

Maximum and minimum water allocated constraint

The allocated water should be between the maximum and minimum water demand:

$$minD(i, j) < X(i, j) < maxD(i, j) \tag{17}$$

Non-negative constraints

Allocated water should be greater than or equal to zero:

$$X(i, j) \geq 0 \tag{18}$$

Finally the model is formulated as follows:
Minimize:

$$OF = [SI \times Gini_{D.A.I}(x) + (1 - SI) \times CVaR(x)] \tag{19}$$

Subjected to:

$$\begin{cases} \sum_{i-1}^{i}\sum_{j=1}^{j} X + EWD \leq E(\tilde{Q}) \\ minD(i, j) < X(i, j) < maxD(i, j) \\ X(i, j) \geq 0 \end{cases} \tag{20}$$

This model was designed only for surface water allocation and changes in constraints would be made if other water resources (including groundwater) are considered. It was also assumed that existing water facilities (such as water transmission lines) would have sufficient capacity. Otherwise, constraints such as technical constraints were added to the model.

Data generation

Due to the limitation of available data and the need to accurately estimate risk factors, the Monte Carlo sampling method was used to generate annual runoff data. This method is based on generating random numbers from statistical distributions (Kalos & Whitlock 2009; Sobol 2018). First, by fitting different statistical distributions to the available data, the distribution which best fits the historical data was found. The analysis showed that the distribution of annual runoff in the study area is the inverse Gaussian distribution. In probability theory, the inverse Gaussian distribution (also known as the Wald distribution) is a two-parameter family of continuous probability distributions with support on (0, ∞). Its probability density function is given by:

$$f(x; \mu, \lambda) = \left[\frac{\lambda}{2\pi x^3}\right]^{\frac{1}{2}} exp\left\{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right\} \quad x > 0 \tag{21}$$

where $\mu > 0$ is the mean and $\lambda > 0$ is the shape parameter (Seshadri 2012). Using the least squares method, the parameters μ and λ were obtained as $1.0372E+8$ and $8.5318E+8$, respectively. Annual flow data values were obtained by generating 10,000 random numbers from the above distribution (see Figure A in the Supplementary Material). Then the annual data were disaggregated to monthly data using the Valencia & Schaake (1973) method. This method uses the following linear relationship to generate monthly data:

$$Y = AX + BV \tag{22}$$

where Y is a (12×1) vector of correlated random variables (monthly data) and X is annual data, A is a (12×1) coefficient vector, V is a (12×1) vector of independently distributed standard normal deviates and B is a (12×12) coefficient matrix (Valencia & Schaake 1973; Kossieris et al. 2018). The parameters of this model can be estimated as follows:

$$\hat{A} = S_{YX}S_{XX}^{-1} \tag{23}$$

$$\hat{B}\hat{B}^T = S_{YY} - \hat{A}S_{XY} \tag{24}$$

where S is a covariance matrix and B is a lower triangular matrix obtained by the principal component analysis. The values of A and B coefficients are presented in Boxes (a) and (b) in the Supplementary Material. To calculate the SI, first the reliability, resilience, and vulnerability (RRV) values were calculated using Equations (10)–(12). Figure 4 shows a Box-Whisker diagram of RRV. The SI value was

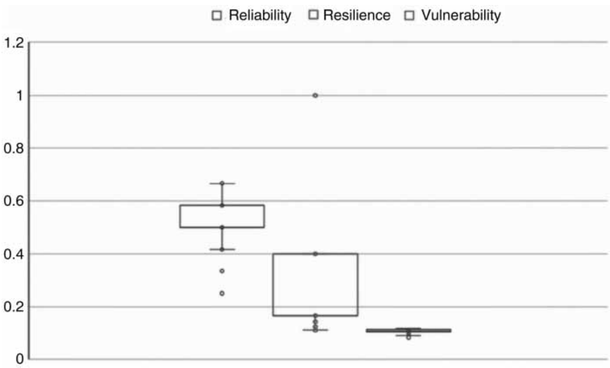

Figure 4 | Reliability, resilience and vulnerability box–whisker diagram.

Table 2 | The results of RRV and SI calculations

Variable name	Mean	Max	Min
Reliability	0.5307	0.6667	0.2500
Resiliency	0.2831	1.0000	0.1111
Vulnerability	0.1083	0.1189	0.0818
$SI = (\text{Reliability})(\text{Resiliency})(1 - \text{vulnerability}) = 0.1300$			

Table 3 | The values and probabilities of annual runoff ($\tilde{Q}$)

N	Values	Probabilities
1	61,266,760.16	0.2787
2	106,876,638.53	0.5915
3	178,209,919.73	0.1273
4	296,336,681.57	0.0025

obtained using Equation (9). Table 2 shows RRV detailed results and the calculated SI.

According to Stroud & Secrest (1966), the Gaussian Quadrature ($N = 4$) method was used to obtain the probabilities and values of the stochastic parameter $\tilde{Q}$ (annual runoff) from the probability distribution function. Table 3 shows the obtained results. In this table the parameter N is the number of probability-value pairs in the discrete approximation. Table 4 shows the parameters used in the developed model.

RESULTS AND DISCUSSION

Since the objective function obtained was a nonlinear function, so the model was a nonlinear model. To solve the model, four scenarios were considered as follows:

1. Determining the weight of objective functions in compromise programming with sustainability index (SI scenario).
2. Considering the Gini function and eliminating the risk function in the objective functions (EQ scenario).
3. Considering equal weight for risk and equality objective functions (EW scenario).
4. Considering the CVaR function and eliminating the Gini function in the objective functions (CVaR scenario).

The outputs of the different scenarios are shown in Table 5. In this table X_{ij} represents the allocation matrix, i represents sectors, including domestic (D), agricultural (A), and industrial (I). Also, parameter j represents sub-basins including the Givi (GI) and Sangvar (SA) sub-basins.

According to Figure 5, the CVaR scenario had the maximum economic benefit. As expected, only the risk function

Table 4 | The parameters used in the developed model

Sector	*B* Givi (dollar/m^3)	*B* Sangvar (dollar/m^3)	*Min D* Givi (10^6 m^3)	*Min D* Sangvar (10^6 m^3)	*Max D* Givi (10^6 m^3)	*Max D* Sangvar (10^6 m^3)
Domestic	0.70	0.65	1.2	2	1.5	2.3
Agricultural	2.50	1.70	4	6	17	34
Industrial	9.46	8.49	14	12.8	41	30
Sub-area	***s*** **(m^2)**	***POP*** **(person)**	***EWD*** **(m^3)**			
Givi	1,560	8,520	6,300,000			
Sangvar	20,820	15,080	4,700,000			
Total	22,380	23,600	11,000,000			

Table 5 | Allocation matrix in four considered scenarios

	SI		EQ		EW		CVaR	
X_{ij}	GI	SA	GI	SA	GI	SA	GI	SA
D	1.395	2.024	5.744	5.245	1.524	3.279	1.290	2.003
A	4.129	6.511	4.542	28.752	4.921	6.356	4.900	6.452
I	14.832	65.324	14.027	34.426	14.003	63.915	66.761	12.848

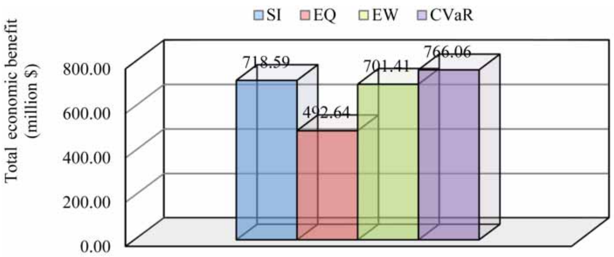

Figure 5 | The total economic benefit comparison.

was optimized in this scenario. The lowest economic benefit was obtained for the EQ scenario, because the EQ scenario puts too much emphasis on equality.

From Figure 6, it can be seen that the minimum and maximum values of the Gini coefficient belong to the EQ and CVaR scenarios, respectively. In the EQ scenario the allocation of resources is based on the equality function. In this case, economic benefit will not be optimal. In the CVaR scenario, the resource allocation is based on the risk function that minimizes the loss. This proves the existence of a trade-off between equality and risk. As a result, a scenario which takes both equality and risk into account should be used. Among these scenarios, the SI scenario will yield the most sustainable, efficient and equitable results because it incorporates all the uncertainties into the model. Based on Figures 5 and 6, it can be shown that the SI scenario has an acceptable result for both economic benefit and equality. Therefore, the SI scenario can be suggested for a decision maker who wants a scenario with maximum economic benefit and equality. Sensitivity analysis is used to evaluate the sensitivity of the model to the *SI* parameter. Sensitivity analysis was performed using the average percentage change (*APC*). The *APC* is calculated as follows:

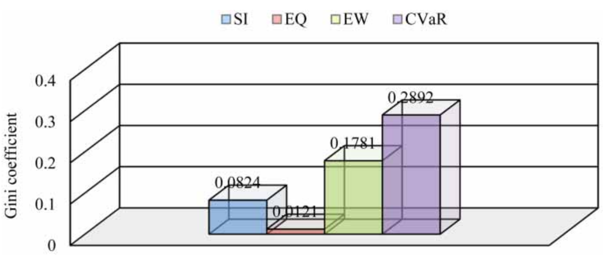

Figure 6 | Comparison of the Gini coefficient for considered scenarios.

$$APC = \frac{1}{i*j}\sum_{i=1}^{i}\sum_{j=1}^{j}\left(\frac{x_{ij} - x_{ij}^*}{x_{ij}}*100\right) \tag{25}$$

where i is the index of the sector, j is the index of the sub-basin, x_{ij} is the model output and x_{ij}^* is the model output due to the *SI* parameter variation (the other parameters were kept constant). Table 6 lists the average percentage change (*APC*) in model output due to the *SI* parameter variation. The obtained data show that the model was highly sensitive to the *SI* parameter.

CONCLUSIONS

Due to the severe water resource limitation, the need for a comprehensive water resource allocation model is clearly evident. This model should not only consider equality and risk factors but it also has to increase sustainability by taking into account existing uncertainties. By considering two objective functions (equality and risk), a new water resource allocation model was developed in this paper. The developed model revealed that the sustainability index (SI) scenario is one of the best scenarios for achieving

Table 6 | The obtained results from sensitivity analysis

Percentage of change in the *SI* parameter	−15	−5	5	15
Average percentage change in model output	−49.0	−50.1	3.0	−18.9

sustainability, together with equality and risk objectives. Some conclusions can be drawn from this study. (1) Although considering the Gini coefficient as an objective function increases the equality, however, without considering the CVaR function, it does not yield good efficiency. Minimizing the CVaR will increase the efficiency by minimizing the loss function. (2) To consider sustainability together with risk and equality, it is suggested to use the sustainability index as the weight of objective functions in a compromise programming approach. Using this index not only makes a trade-off between risk and equality but also creates a comprehensive model. In this model, the SI incorporates the inherent characteristics of the water resources system into the model, and it helps decision makers to allocate water more efficiently and equitably. (3) The CVaR function considers only the uncertainty of the water supply, whereas the SI considers the uncertainties caused by water supply and water demand simultaneously. Therefore, considering CVaR with SI will create a more comprehensive model.

SUPPLEMENTARY MATERIAL

The Supplementary Material for this paper is available online at https://dx.doi.org/10.2166/ws.2020.025.

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