Uncertainty based budget allocation of wastewater infrastructures' flood resiliency considering interdependencies

Mohammad Karamouz and A. Hojjat-Ansari

ABSTRACT

Wastewater treatment plants (WWTPs) are at high risk of coastal flooding due to the proximity of water bodies. Furthermore, power outages caused by extreme weather make their recovery process more challenging. In this paper a multi criterion decision-making (MCDM) method is utilized to involve different aspects of WWTPs' flood resilience into the framework developed for distributing financial resources among them. The flood resilience improvement of plants is investigated for a wide range of interventions and related costs. On this basis, utility of the allocations is developed for each plant and a fixed budget is distributed among them in different ways. Due to the stochastic nature of floods, the uncertain resilience attributes of systems and the subjective views of experts and decision-makers, uncertainties are incorporated into this process. Accordingly, the uncertainty of each plant is targeted. The results show that: water-energy interdependence plays a significant role in assessing the flood resilience of WWTPs; using different ways of allocation leads to varying degrees of uncertainty in investment in each plant; and evidence theory is an effective way of integrating experts' beliefs in the process of allocating resources. The proposed methodology can be implemented on similar cases in different geographic settings.

Key words | flood resilience, resource allocation, uncertainty, wastewater treatment plants, water-energy interdependencies

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INTRODUCTION

Flood is one of the most devastating natural disasters that leads to significant economic loss and other casualties. Historical evidence indicates that coastal cities are extremely vulnerable to floods and storms (McGranahan *et al.* 2007; Kantamaneni 2016; Brown *et al.* 2018). More importantly, critical infrastructures in these areas are usually a flood target. Of these infrastructures, wastewater treatment plants (WWTPs) are usually very close to coastlines to discharge their effluent into receiving waters. Hence, they are among the most vulnerable infrastructures to coastal flooding (Hummel *et al.* 2018). On the other hand, any functional failure in these systems results in irreparable environmental damage which, in turn, places more emphasis on their flood resilience improvement. This improvement is not facilitated except through allocating resources and making investment. Resilience resource allocation needs a framework to help decision-makers to optimally decide how to invest in their systems. Creating such a framework requires the assessment of the system's resilience and the consideration of different interventions and related costs (MacKenzie & Zobel 2016).

The concept of resilience was originally introduced by Holling (1973) in the early 1970s. However, this concept has recently found a new application in urban resilience such that Karamouz *et al.* (2012) consider it the most important risk management indicator. Resilience concept varies in different scientific disciplines (Ayyub 2014). In engineering, resilience is the capacity of a system to adapt to any stressed

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situations (Alderson *et al.* 2015). Specifically, flood resilience is defined as the ability of resisting against flood, absorbing the related negative impacts, and returning to normal condition (Simonovic & Peck 2013). With such a definition, many researchers have provided formulations and frameworks for measuring the resilience of urban communities and infrastructures. One widely used framework has been proposed by Bruneau *et al.* (2003) that is based on quantifying system's resilience dimensions including rapidity, robustness, resourcefulness, and redundancy (four Rs). Rapidity is the ability of a system to achieve the expected level of performance in the shortest possible time while robustness is the system's resistance to maintain its function against stressors. On the other hand, resourcefulness is defined as the ability to employ material and human resources to meet the expected level of service and redundancy is the units that are available for replacement in different parts of the system to ensure its continued operation. On this basis, Kendra & Wachtendorf (2003) investigated the reconstitution of Emergency Operations Centre (EOC) after the destruction in the World Trade Center attack. A conceptual framework based on four Rs was developed by Cutter *et al.* (2008) for understanding the disaster resilience of communities. Karamouz *et al.* (2016) utilized four Rs to quantify the flood resilience of WWTPs against coastal flooding.

From the resilience standpoint, various systems should be prioritized to distribute resources among them. In other words, the more a system has the capacity for resilience improvement, the more investment is justified in that system. To accomplish this a priority multi-criterion decision making (MCDM) technique has been extensively used. Ronco *et al.* (2015) employed the MCDM technique to develop flood risk maps in Switzerland based on ranking the different dimensions of risk. Karamouz *et al.* (2019) utilized MCDM in two ways, to rank flood mitigation alternatives and to assess the flood resilience of WWTPs. By identifying the flood resilience attributes of WWTPs and taking their relative importance into account, Karamouz *et al.* (2018) established a resilience evaluation framework based on the MCDM technique. The flood resilience of WWTPs were quantified in terms of four Rs in this framework to examine the priority of these facilities in the allocation of resources.

Recent flood events such as Hurricane Katrina and Superstorm Sandy in the USA have demonstrated the role of interdependencies in cascading failures (Leavitt & Kiefer 2006; Comes & Van de Walle 2014; Sharkey *et al.* 2015). Interdependency is a linkage connecting two infrastructures in a way that the state of one infrastructure influences the state of the others (Rinaldi *et al.* 2001). This issue was first reviewed by the US Commission on critical infrastructure protection (Ellis *et al.* 1997) and then studied in a growing body of investigations. Wastewater treatment plants are very energy demanding and for normal operation they rely on energy infrastructures. During Superstorm Sandy critical energy infrastructures were damaged by flooding. The storm, for example, caused serious damage to generation, transmission, substation and distribution systems (Bloomberg 2013). Subsequently, some treatment plants lost their power for days, forcing operators to partially operate on emergency generators. Therefore, the reliability of utility power and back-up generators needs to be considered for assessing WWTPs' flood resilience (Hyland *et al.* 2013). If there is no such consideration, resilience tends to be overestimated or underestimated and leads to wrong judgments.

The motivation behind quantifying resilience is usually to assist decision makers to prepare for and respond to interruptions in the systems (MacKenzie & Zobel 2016). This preparation is typically done by distributing physical and financial resources among infrastructures to mitigate the impacts of disruptive events. However, due to budget limitations, the optimal allocation of these resources is inevitable (Zhang *et al.* 2018). The optimal allocation of flood resilience resources among different facilities of an agency is difficult for two reasons: (1) because of the nature of resource allocation problems in which the interest of one facility may work against the others (Chakraborti *et al.* 2015); and (2) because of the uncertainties that exist in measuring and improving their flood resilience (Berkes 2007). The latter is the main reason for investors' unwillingness to invest in systems' resilience enhancement (Juan-García *et al.* 2017). Therefore, it is important to better understand the uncertainties in order to inform agencies of their investment risk.

In distributing flood resilience resources, uncertainties are due to the stochastic nature of floods and the uncertain resilience attributes of systems. Furthermore, due to the subjective views of experts and decision-makers, another set of uncertainties is entered into the process. Agarwal *et al.*

(2004) categorized the earlier and the latter into stochastic (aleatory) and subjective (epistemic) uncertainties, respectively. The stochastic uncertainty stems from the inherent randomness of events and cannot be reduced while the subjective uncertainty arises due to insufficient knowledge and can be reduced. Stochastic uncertainty is typically analyzed using probability theory. Karamouz *et al.* (2016) utilized probability theory and the Monte Carlo technique to estimate stochastic uncertainty in measuring the flood resilience of WWTPs.

In respect to policy making, the extent of uncertainty obviously includes subjectivity and different perspectives of various actors involved in the decision making process (Walker *et al.* 2012). Specifically, in resource allocation usually a limited number of experts offer their opinions based on their limited knowledge and information (Ballent *et al.* 2019). In addition, in this process, the experts cannot easily express their judgements on the available options with exact and crisp interpretations (Ju & Wang 2012). It is therefore important to combine the views of experts to estimate the subjective uncertainty in the allocation of resources. Conventional probability theory is unsuitable to represent subjective uncertainty (Ali *et al.* 2012). Instead, the Dempster-Shafer theory (Shafer 1976) that is also called evidence theory makes it possible to deal with such problems and to combine information coming from independent sources (Ju & Wang 2012). This theory offers a degree of belief as a measure of an expert's belief over a proposition (Vick 2002). Many researchers such as Yang & Xu (2002), Yang *et al.* (2006), and Guo *et al.* (2007), have utilized the Dempster–Shafer theory for developing an evidential reasoning algorithm for MCDM in decision-making and selection processes. In this paper, the Dempster–Shafer theory is utilized as a post processor for evidential reasoning in the resource allocation process.

Such an approach increases decision confidence in the allocation of resources.

In recent years, the resilience of urban infrastructures to natural hazards has been the focus of many investigations. However, as could be observed in the literature review, the way resilience is measured and resources are allocated among multiple facilities have received limited attention. In many flood disasters, disruption in energy/electrical resources is the key factor in systems shutdown as observed in the case of Superstorm Sandy in NYC. Furthermore, many urban facilities are run by an agency that has the responsibility for funds allocations. It is often done through a non-technical platform with a lack of a performance/growth potential base matric. Some previous studies realized that it is done on technical ground without combining it with many subjective evidences and information and the uncertainties associated with them as well as technical attributes. These shortcomings have been realized in the previous works of the authors (Karamouz *et al.* 2016, 2018). However, their studies are significantly extended to include energy dependencies in evaluating the coastal flood resilience of waste water infrastructures; and different allocation methods including maximizing overall resilience (MOR), and examining uncertainties in the financial allocation process by having a better way of assigning utility functions to different facilities. The new contribution of this paper is, however, to develop a framework extending the application of Nash product (MNP) and consumer behavior theory (CBT) in order to further close the gap between how agencies allocate funds and how decision making tools combining different analyses and different views could be utilized.

The remainder of the paper is organized as follows. In the next section, methodology is described in detail. In this section, different dimensions of WWTPs' flood resilience and their improvement through investment will be addressed. Also, the allocation of financial resources among these facilities will be examined in different ways. Next, a case study is presented. The results of uncertainty based allocations are illustrated and discussed followed by a summary and conclusion.

MATERIALS AND METHODS

Resilience assessment is generally classified into one of two categories: attribute based (qualitative) and performance based (quantitative) assessments (Hosseini *et al.* 2016; Vugrin *et al.* 2017). Performance based measuring of resilience requires system's simulation during hazards. With the increasing number of systems and their complexity, the simulation has become more and more difficult. In particular, consideration of systems' interdependencies increases the simulation time burdens. However, it provides a detailed description of systems' performance during hazards. In comparison, in the evaluation of systems' resilience based on their attributes, there is no need to simulate systems. This approach is a low-cost technique to compare the systems' resilience with a minimum degree of complexity in resource allocation problems. Thus, this paper extends the work of Karamouz *et al.* (2016, 2018) at using an attribute based approach to assess the resilience of WWTP systems through four criteria including rapidity, robustness, resourcefulness and redundancy (four Rs) and sub-criteria that include different data types, such as hydrological, environmental, economic, and technical data. Also, given the importance of energy for the operation of sewage treatment systems and the possible failure of electrical infrastructures during floods, the relevant sub-criteria are identified and incorporated into the framework. The proposed framework is illustrated in Figure 1. According to this figure, the framework consists of four steps:

1. Identifying the attributes of systems that are effective in flood resilience.
2. Comparing the flood resilience of systems using MCDM.
3. Investing in systems and examining their flood resilience improvement.
4. Allocating a fixed budget among systems considering uncertainties.

Each of these steps will be described in detail.

System's characterization

Among different WWTPs' flood resilience attributes, 23 attributes, 18 of which were identified by Karamouz *et al.* (2019), are utilized for evaluating the system's flood resilience. These sub-criteria are grouped into four Rs according to Table 1. Selection of these sub-criteria has been done to cover a wide range of different situations of WWTP in normal (such as the design capacity of plant), emergency (such as the number and capacity of backup generators) and recovery conditions (such as recovery time). The expert assessment and weighting methodology are also involved based on these physical processes and site specific characteristics as explained in the case study section. Some more details are presented in the supplementary material of this paper. Steps 1–3 of the flow chart are extension of previous works by adding new energy sub-criteria and new financial utility function. Step 4 is the main contribution of this paper in this paper.

Interdependencies

One important aspect which is often neglected in examining systems' resilience is interdependencies among them. Sewage treatment systems need electricity to operate. This necessity becomes more acute in flood situations when high flow is entered into the systems. However, there is always a high probability of experiencing power outages at the time of severe storms. Wastewater treatment plants are usually equipped with back-up power to support their performance in emergency situations. However, due to the high cost of back-up generators, the total electrical power capacity required is not provided. Therefore, the number and capacity of generators in critical situations play a significant role in the rapid recovery of them. Furthermore, the safety of substations inside the plants and electrical feeders against floods should also be considered. Because of the high inundation depth at the unit substations, main plant switchgear, and transformers may also result in complete shutdown of the plants. Based on historical observation of disruption to electricity connected to wastewater treatment plants, their flood resilience has also been assessed. This number is set to zero and one, where one indicates disruption during a storm. All the above-mentioned factors are specified into five sub-criteria in Table 1 and are assumed to represent water–energy interdependencies. These factors are bolded in this table besides 18 other sub-criteria defined by Karamouz *et al.* (2019).

Resilience quantification

Using the values of sub-criteria and their respective weights, the attribute-based resilience for any of the WWTPs can be calculated. For this purpose, different approaches based on MCDM can be used. One of the recent approaches that has been successfully used is the Promethee and Gaya method (Behzadian *et al.* 2010). The relationships of this method, that is explained in detail in Karamouz *et al.* (2016), are presented in the supplementary material.

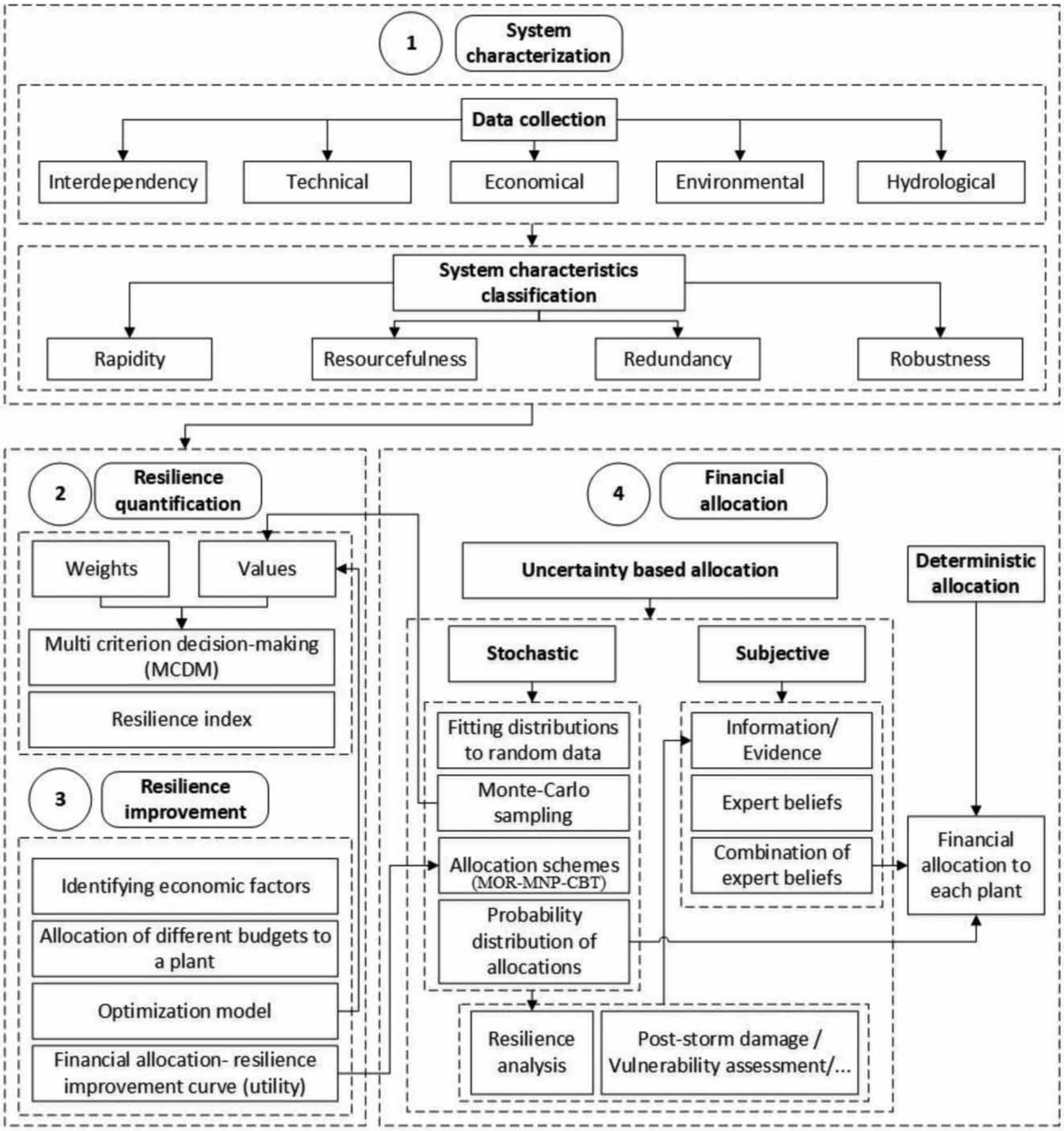

Figure 1 | Proposed framework for allocating financial resources to improve WWTP flood resilience.

Resilience improvement

Here, the optimal improvement of system's resilience for a given budget is of interest. In doing so, economic factors are identified among the sub-criteria. These factors include items that potentially can be improved by investment. These factors are listed in Table 1A in the supplementary material. In this table, all the costs are presented in 2013 US dollars and are averaged over the range of available costs. In the next step, allocation constraints are determined for each economic sub-criterion (ESC) in an optimization problem. Then, the resilience improvement of each plant is evaluated for different budgets. Accordingly, a financial allocation-resilience improvement curve (hereafter utility function) is developed for each plant that is further used as their utilities of the received budget in the financial allocation process.

Table 1 | Different sub-criteria for WWTPs to quantify flood resiliency (adopted from Karamouz *et al.* (2016)

Criteria	ID	Sub-criteria description	Unit
Rapidity	Ra_1	Hurricane flood elevation (based on North American Vertical Datum of 1988 (NAVD88))	Ft
	Ra_2	Adverse environmental impacts on the surrounding area (due to treatment failure caused by flooding)	–
	Ra_3	Plant design capacity	MGD[a]
	Ra_4	Post-stress recovery	Hour
	Ra_5	Population served (number of users served by the plant)	#
	Ra_6	Untreated or semi-treated effluent discharge	MG
	$\mathbf{Ra_7}$[b]	Average electricity consumed by the plant	KWH[c]
	$\mathbf{Ra_8}$[b]	Plant back-up generator capacity	KWH
Robustness	Ro_1	Additional load in time of flooding (the difference between WWTP capacity for the total maximum wet and dry weather flow. Maximum wet weather flow is the maximum flow received during any 24-hour period. Maximum dry weather flow is the maximum daily flow during periods without rainfall)	MGD
	Ro_2	Critical flood elevation (100-year flood elevation +30 inches for expected sea level rise by the 2050s, which is determined based on the Federal Emergency Management Agency's new advisory base flood elevation maps for a 100-year flood event, was selected as the baseline for the analysis)	ft[d]
	Ro_3	Maximum inundation depth (due to the flat terrain of the plant, several areas may be flooded by up to this value of water during the critical flood event)	ft
	Ro_4	Percent of not-at-risk equipment (percent of plant items that are not at risk of damage during flood)	%
	Ro_5	DMR violations (the percentage of discharge monitoring reports that resulted in effluent violations. During minimal levels of stress, the DMR violation percentages are indicative of how well each treatment plant can cope with daily operational stresses)	%
	Ro_6	Damage cost from the most severe historical hurricane (without flood protection for the plant)	$
	$\mathbf{Ro_7}$[b]	100-year flood inundation depth at unit substation	ft
	$\mathbf{Ro_8}$[b]	Experiencing power loss during storms	–
Resourcefulness	Rs	Number of plant technical staff	#
	Rs	Availability of dewatering facilities (facilities to drain sludge to decrease 90% of its liquid volume)	–
	Rs	Total risk avoided for every single dollar spent over 50 years	$
Redundancy	Rd_1	Existence of underground tunnel systems	–
	Rd_2	Availability of WWTPs in the neighboring areas (distance from the closest WWTP)	km
	Rd_3	On-site storage (volume of lakes in the WWTP's zone)	ft^3
	$\mathbf{Rd_4}$	Number of back-up generators	#

[a]Million gallons per day (US liquid gallon = 3.78 L).
[b]Sub-criteria representing water-energy interdependencies.
[c]Kilowatt hour.
[d]Foot = 0.3048 m.

Optimization model

After identifying economic sub-criteria and their allocation constraints, an optimization is performed to maximize the resilience increase of the *i*th system for a total budget, TB, as follows:

$$\max \Delta R_i = R_i(ESC_1.\ ESC_2.\ \cdots.\ ESC_{NE}) - R_i(\widehat{ESC}_1.\ \widehat{ESC}_2.\ \cdots.\ \widehat{ESC}_{NE}) \quad (1a)$$

subject to:

$$ESC_j = \begin{cases} \widehat{ESC}_j + [Alloc_2/M_j] & \text{if } j = NE \\ \widehat{ESC}_j + Alloc_j/M_j & \text{otherwise} \end{cases} \quad (1b)$$

$$\sum_{j=1}^{NE} Alloc_j = TB \quad (1c)$$

$$Alloc_j \geq 0 \quad j = 1.2.\ \ldots.\ NE \quad (1d)$$

$$Alloc_1 \leq \alpha TB \quad (1e)$$

$$Alloc_2 \leq M_2(Ra_7 - Ra_8) \quad (1f)$$

$$Alloc_3 \leq \beta TB \quad (1g)$$

$$Alloc_4 \leq \gamma TB \quad (1h)$$

$$Alloc_5 = \begin{cases} M_5 & \text{if } Rs_2 = 0 \\ 0 & \text{otherwise} \end{cases} \quad (1i)$$

$$Alloc_6 \leq \lambda TB \quad (1j)$$

where ΔR_i is the resilience improvement after any investment in the ith plant, which is calculated as the difference between the improved resilience and the initial resilience. R_i is determined according to Equation (4) in the Appendix. $Alloc_j$ is the part of TB goes to the jth economic sub-criterion, ESC_j, and $\widehat{ESC}_j$ is the value of ESC_j before the investment. Ra_7, Ra_8, and Rs_2 are the values of corresponding sub-criteria for each plant (see Table 1). M_j is the cost corresponding to the unit increase of the jth ESC according to Table 1A in the Appendix and NE is the number of economic sub-criteria.

The resilience improvement of a plant is maximized using genetic algorithm. This algorithm is repeated for different budgets (i.e. $10, $50, $100, $200, and $300 million) until the resiliency improvement rate is insignificant) Also, there is no need to extend the budgets to draw allocation-resilience improvement curves, given the $187 million budget that is distributed among the plants to compare with the NYCDEP's (the organization responsible for the operation of sewage treatment systems serving NYC) allocation. Although repeating the algorithm for more budgets in this interval provides more points to derive the functions of these curves, these five budgets seem sufficient to also compare the results with Karamouz *et al.* (2018). Equation (1b) shows the incremental improvement of sub-criteria values after the allocation of budgets. The budget allocated to increase a plant's treatment capacity and on-site storage is defined at a maximum of αTB (Equation (1e)). When the generator capacity is less than the power required by the plant, the capacity will be increased depending on the allocated budget to finally supply the power needed (Equation (1f)). WWTPs are typically equipped with 1,000–2,000 KW generators (NYCDEP 2007), thus, based on Table 2A in the supplementary material, it is assumed that for every 2,000 KW addition, one unit is added to the number of generators, which is equivalent to 1.6 million dollars. This amount of money has already been included in the allocation to the generators' capacity. The allocation limit on sand bagging, water proofing, and elevating critical equipment is assumed based on Equation (1g) up to βTB.

Table 2 | Inference from evidence on financial allocation proposition

[Bel Pl]	Inference from evidence
[1.0 1.0]	Completely true
[0.0 0.0]	Completely false
[0.0 1.0]	Absolutely unknown
[Bel 1.0]	Tends to be supported
[0.0 Pl]	Tends to be refuted
[Bel Pl]	Tends to be supported or refuted

The upper bound of increasing the number of technical workers is γTB (Equation (1h)). Equation (1i) indicates that the budget allocated to a plant on dewatering facilities is $7 million ($M_5 = 7$ from Table 2A in the supplementary material) provided that the plant does not have these facilities. Otherwise, no fund will be allocated to this section. The allocated budget limit to increase the on-site storage of plants is defined up to λTB (Equation (1j)). The coefficients of α, β, γ, and λ represent the percentage of TB, which are defined by the experts of each plant. It should be noted that these constraints are defined based on preliminary assessment. In practice, more rigorous analysis can be carried out by the experts of each system to better define these constraints.

Budget allocation

In this study, three methods are investigated to allocate financial resources among systems. The first method is to maximize overall resilience improvement and the allocations to the plants is geared toward this goal. In the second method, considering systems' utilities of the received budgets, the Nash bargaining solution is used and the financial allocation is performed based on maximizing the Nash product. Finally, the allocation problem is solved using consumer behavior theory. In the following sections, each of these methods will be discussed.

Maximizing overall resilience (MOR)

In this approach, the allocation among systems is carried out in the form of an optimization problem. Based on the resilience improvement of each system for the received allocations, a fixed budget is distributed among them with the aim of maximizing the resilience increase of all systems as follows:

$$\max \; Z_1 = \sum_{i=1}^{N} \Delta R_i \tag{2a}$$

$$subject\ to: \; \sum_{i=1}^{N} AB_i = TB \tag{2b}$$

$$AB_i \leq AB_i^* \tag{2c}$$

$$AB_i \geq 0 \tag{2d}$$

where ΔR_i and AB_i are the resilience increase and the allocated budget for the ith wastewater treatment plant, respectively. AB_i^* is the allocation at which the rate of resilience increase of the ith system becomes less than 0.01. The values of AB* are obtained by the optimization model run for each plant in the previous section. In Equation (2c), it is assumed that the allocation more than AB_i^* is not cost-effective any more. *TB* is the total budget intended to be allocated to all systems and N is the number of facilities.

Maximizing Nash product (MNP)

In this approach, the allocation-resilience improvement curves are considered to be the utility of each plant for the allocated budget. As allocations increase the resilience of a system, the utility of the system will increase proportionally. Here, the utility is defined as the plants' resilience improvement of the allocated budgets. Assume that N is the number of systems anticipating resource allocation. Let $U = (U_1, U_2, \ldots, U_N)$ be the vector of systems' utility functions. These functions are based on the optimization model for each plant separately and fitting a logarithmic function to the incremental budget increase against the percentage of resilience improvement. The unique solution for the allocation problem based on maximizing Nash product is as follows:

$$\max \; Z_2 = \prod_{i=1}^{N} (U_i - t_i) \tag{3a}$$

$$subject\ to: \; U_i > t_i \tag{3b}$$

where $t = (t_1, t_2, \ldots, t_N)$ is the disagreement vector. In practice, each plant manager can determine the minimum amount of funding it needs as the point of disagreement in the allocation process. Other constraints are defined similar to Equations (2b) and (2c).

Consumer behavior theory

The theory of consumer behavior is based on the concept of marginal utility (Michael & Becker 1973). In this study, marginal utility is defined as the additional resilience improvement derived from expending an additional dollar on wastewater treatment plants. This theory involves the application of two laws: the law of diminishing marginal utility and the law of equi-marginal utility (Michael & Becker 1973). According to the first law, the additional utility derived from an additional dollar spend on each plant needs to be decreasing. The latter explains how NYCDEP should distribute a fixed budget among its plants to maximize its total utility in such a way that its marginal utility of the last dollar spent on each plant becomes equal. Therefore, the optimum allocation by which the total utility of NYCDEP being maximized is obtained by solving a system of equations as below:

$$\frac{MU_i}{AB_i} = \frac{MU_j}{AB_j} \quad \forall i.\, j \tag{4a}$$

$$MU_i = \frac{\partial U_i}{\partial AB_i} \tag{4b}$$

where MU_i and MU_j represent the marginal utility of allocation to the ith and jth plants, respectively.

Uncertainty analysis

Uncertainties in the financial allocation process to improve the flood resilience of wastewater treatment plants are divided into two categories:

1. Uncertainties due to the random nature of flood and the uncertain flood resilience attributes of plants (stochastic uncertainty).
2. Uncertainties due to subjective views of experts and decision-makers (subjective uncertainty).

Therefore, the financial allocation to each plant has some degree of uncertainty. These uncertainties are discussed in the following sections, respectively.

Stochastic (objective) uncertainty

Here, the Monte Carlo technique is applied to incorporate the uncertainty arising from the randomness of flood and the resilience attributes of systems into the financial allocation process. This approach is a class of computational algorithms that rely on random sampling to represent uncertainty. In this regard, the sub-criteria that are random in nature are identified. In the next step, for each random sub-criterion, an appropriate statistical distribution is fitted to the records of all plants for the respective sub-criterion. These sub-criteria are listed in Table 2A of the supplementary material with their distribution types and parameters in the Appendix. Based on the random samples produced by Monte Carlo simulation for these sub-criteria, 100 utility functions are generated for each plant, and are randomly assigned in each allocation process. With a thousand iterations ($I = 1{,}000$) of allocation process, an allocation series was obtained for each plant. Assuming that the allocation series for a plant is normally distributed, the probability distribution (pdf) and the cumulative distribution of allocation is developed and the 95% confidence interval of allocation is determined for each plant (Figure 2). Finally, the best allocation option is selected by maximizing Z_3 and Z_4 instead of Z_1 and Z_2, respectively. In other words, the expected value of MOR and MNP are maximized in this case.

$$\max \quad Z_3 = \sum_{k=1}^{I} \sum_{i=1}^{N} \Delta R_{i.k} \tag{5}$$

$$\max \quad Z_4 = \sum_{k=1}^{I} \prod_{i=1}^{N} U_{i.k} \tag{6}$$

where $\Delta R_{i.k}$ and $U_{i,k}$ are the resilience improvement and the utility of the ith plant in the kth iteration, respectively.

Subjective uncertainty

In the process of allocating funds, an agency should consider different views of its experts. Based on their information and knowledge, these individuals will reach a degree of agreement on the level of allocation to each system. Such a degree of belief can be achieved in different ways including resilience analysis, post-storm damage assessment, building and infrastructure-level vulnerability assessment, etc. Combining the degree of expert beliefs is inevitable to avoid unilateral decisions and judgments. For this purpose, the financial allocation ranges are categorized into the types of low, medium and high (Figure 3).

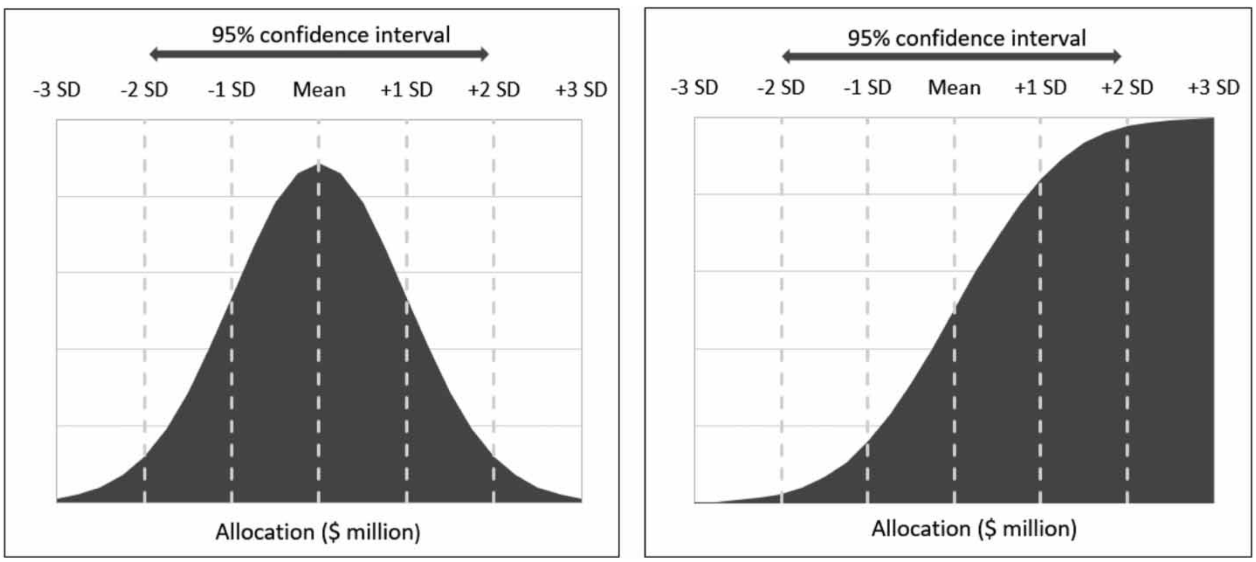

Figure 2 | Probability distribution function (left), cumulative distribution function function (right), and 95% confidence interval of allocation as a function of the standard deviation (SD) [–1.96SD – 1.96SD].

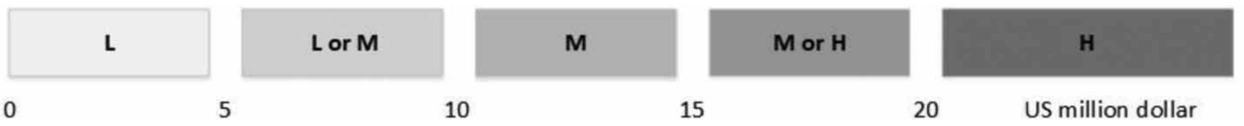

Figure 3 | Financial allocation categories.

For example, the allocation of less than \$5 million could be considered as the low allocation (L), \$10–15 million as the medium (M), and more than \$20 million as the high (H). The allocation of \$5–10 million could be considered as either of low or medium while the allocation between \$15 and \$20 million could be associated with either medium or high. On this basis, the probability that the allocation to the ith plant is placed in each of these intervals can be determined as follows:

$$Pr(a \leq AB_i \leq b) = \int_a^b pdf(AB_i)dx \tag{7}$$

where a and b are the lower and upper bounds of each interval. $Pr(a \leq AB_i \leq b)$ represents the probability that the allocated budget to the ith plant, AB_i, is placed between a and b. Further, uncertainty over each interval of allocation is evaluated by the theory of evidence.

Assume that $\theta = \{L.\ M.\ H\}$ is the universal set of allocation types. $\Omega(\theta)$ which represents a power set of θ is defined as follows:

$$\Omega(\theta) = \{\phi.\ \{L\}.\ \{M\}.\ \{H\}.\ \{L.M\}.\ \{L.H\}.\ \{M.H\}.\ \{L.\ M.\ H\}\} \tag{8}$$

Each element of $\Omega(\theta)$, E, represents a proposition about the type of allocation to a plant. The beliefs of experts in each of these propositions are mapped into [0 1] by a function called belief mass function, $m()$. The belief mass function of each proposition, $m(E)$ shows the contribution of that proposition to the evidence. This function must satisfy the axioms below:

$$m(E) \geq 0 \quad \forall E \in \Omega(\theta) \tag{9a}$$

$$m(\varnothing) = 0 \tag{9b}$$

$$\sum m(E) = 1 \tag{9c}$$

Expert's belief mass over $\{L, H\}$ is zero ($m(\{L, H\}) = 0$), because the propositions of low and high do not have any overlaps. Since it is assumed that there is no expert's ignorance, the belief mass over $\{L, M, H\}$ is also zero ($m(\{L, M, H\}) = 0$). Hence, these propositions can be eliminated from the set of $\Omega(\theta)$. For the other propositions, the mass of belief is assumed to be proportional to the probability distribution of allocation to each plant.

Let $m_1, m_2, \ldots, m_n$ be the belief masses of n experts, their joint belief mass over the proposition $\hat{E}$, $\mathrm{m}(\hat{E})$, can be calculated as follows:

$$\mathrm{m}(\hat{E}) = \frac{1}{1-k} \sum_{\bigcap_{i=1}^{n} E_i = \hat{E}} m_1(E_1)\, m_2(E_2) \cdots m_n(E_n) \tag{10}$$

where k represents conflicts among the belief masses of experts. The large value of k indicates high inconsistency among different expert beliefs. $\frac{1}{1-k}$ is the normalization factor to completely ignore the conflicts among belief masses and attribute it to the null set, ϕ. The inconsistency, k, is determined as follows:

$$\mathrm{k} = \sum_{\bigcap_{i=1}^{n} E_i = \varnothing} m_1(E_1)\, m_2(E_2) \cdots m_n(E_n) > 0 \tag{11}$$

After derivation of joint masses, belief and plausible intervals for each proposition of allocation are determined for the plants. The belief function represents the lower limit of probability over $\hat{E}$, which is defined as follows:

$$Bel\colon \Omega(\theta) \rightarrow [0 \ \ 1]$$

$$Bel(\hat{E}) = \sum_{E \subset \hat{E}} m(E) \tag{12}$$

Therefore, the belief over each proposition of financial allocation can be calculated through Equations (13a)–(13e):

$$Bel(\{L\}) = m(\{L\}) \tag{13a}$$

$$Bel(\{L.\ M\}) = m(\{L\}) + m(\{L.\ M\}) + m(\{M\}) \tag{13b}$$

$$Bel(\{M\}) = m(\{M\}) \tag{13c}$$

$$Bel(\{M.\ H\}) = m(\{M\}) + m(\{M.\ H\}) + m(\{H\}) \tag{13d}$$

$$Bel(\{H\}) = m(\{H\}) \tag{13e}$$

In contrast, the plausibility function indicates an upper limit of probability over $\widehat{E}$ and is defined as follows:

$$Pl: \Omega(\theta) \rightarrow [0 \quad 1]$$

$$Pl(\hat{E}) = \sum_{E \cap \widehat{E} \neq 0} m(E) \tag{14}$$

Accordingly, the plausibility over each proposition of allocation can be calculated through Equations (15a)–(15e):

$$Pl(\{L\}) = m(\{L\}) + m(\{L.\ M\}) \tag{15a}$$

$$Pl(\{L.\ M\}) = m(\{L\}) + m(\{L.\ M\}) + m(\{M\}) + m(\{M.\ H\}) \tag{15b}$$

$$Pl(\{M\}) = m(\{L.\ M\}) + m(\{M\}) + m(\{M.\ H\}) \tag{15c}$$

$$Pl(\{M.\ H\}) = m(\{L.\ M\}) + m(\{M\}) + m(\{M.\ H\}) + m(\{H\}) \tag{15d}$$

$$Pl(\{H\}) = m(\{H\}) + m(\{M.\ H\}) \tag{15e}$$

Furthermore, the uncertainty over each proposition of allocation can be illustrated as [*Bel Pl*] based on Figure 4 and its negation or disbelief interval (*Dl*) is calculated as follows:

$$DI(\hat{E}) = 1 - Pl(\hat{E}) \tag{16}$$

Based on the values of belief and plausibility, one can interpret the uncertainty over each proposition of allocation. If the degree of belief over a proposition is 1, the evidence absolutely supports the proposition, and the proposition is quite true. In case of zeroing belief and plausibility, it is ensured that the intended statement about the allocation is quite false. If the degrees of belief

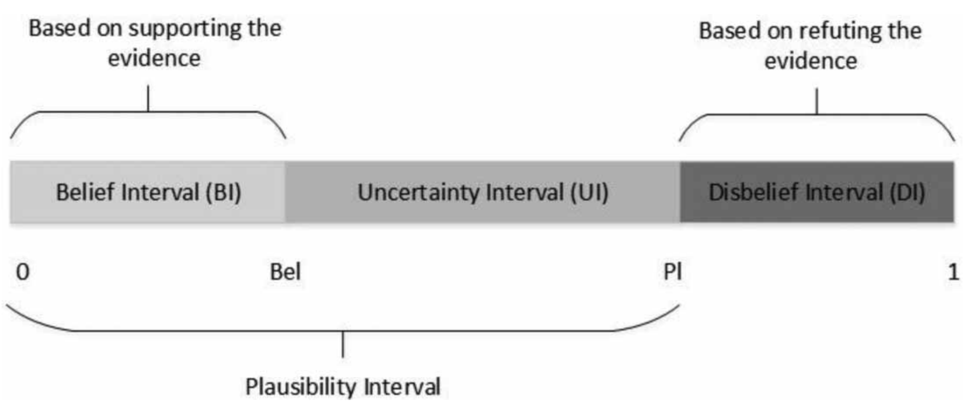

Figure 4 | Belief (Bel), plausibility (Pl), and uncertainty intervals of financial allocation (Bae *et al.* 2003).

and plausibility become 0 and 1, respectively, the evidence about the proposition is absolutely unknown. By increasing the degree of belief, the correctness of the proposition increases and vice versa. The above interpretations of Figure 4 are summarized in Table 2.

CASE STUDY

New York City's wastewater treatment system is at high risk of inundation caused by hurricanes and tropical storms and is selected to illustrate the methodology. The city's Department of Environmental Protection (NYCDEP) is in charge of the entire operation and maintenance of the sewage treatment system. The NYCDEP wastewater treatment system includes 14 plants that discharge their effluent into the receiving water surrounding the city. These facilities together process over 1.3 billion US gallons (nearly five million cubic meters) of wastewater daily. After Superstorm Sandy, the insufficient flood resilience of the plants was perceived by the NYCDEP. Accordingly, the agency's study acknowledged that all fourteen wastewater treatment plants are at risk with a consequence of over $900 million against a 100-year flood. In this respect, the recommended budget to improve the flood resilience of these facilities was estimated at $187 million (NYCDEP 2013). This budget mainly includes the cost of elevating critical equipment, flood proofing, sand bagging, and constructing barriers.

Figure 5 illustrates the plants' names, locations, capacities, and their service area boundaries on the NYC map. According to this figure, some of these treatment plants are directly exposed to surges in the Atlantic Ocean, such as Coney Island and Rockaway. Some of them are exposed after passing through Long Island Sound, such as Hunts Point, and therefore part of the storm surge is dissipated. Others are exposed by the propagation of surges in the rivers, such as the North River plant. These site-specific exposures to hazards are considered in assigning values to the sub-criteria in Table 3.

Table 4 shows the proposed coefficients of financial constraints for these plants based on expert advice. Given the considerable budget required for increasing the plants' resilience against coastal flooding, efforts to optimally distribute this budget among them are of great importance. In the next section, the results of such efforts are presented.

RESULTS

The weights assigned by the experts are combined and the relative importance of each factor is determined using an analytical hierarchy process (Saaty 1990). The weights of criteria and sub-criteria are presented in Table 5. As illustrated, the weights of four Rs are not the same. Rapidity and robustness are assigned higher weights with 0.395 and 0.349, respectively. This is because the resilience of a system mainly results from the inherent strength and characteristics to withstand flood and the speed of the system to recover (Bruneau *et al.* 2003; Karamouz *et al.* 2016; Ouyang 2017). However, resilience can also be improved by means of resourcefulness and adding redundant elements to the system. Hence, all the elements of the four Rs are important and need to be counted in quantifying resilience.

The flood resilience of NYC's wastewater treatment plants based on MCDM (Equation (4a)) is illustrated in Figure 6. The Red Hook plant has the highest amount of resilience among others. This plant did not experience major flooding and preserved its basic performance during Superstorm Sandy. In contrast, the Rockaway and Coney Island plants have the least amount of resilience. Both of these plants experienced major flooding during Sandy and many of their electrical equipment were submerged by the flood (NYCDEP 2013). In the Rockaway plant, for example, both utility feeders serving the plant were interrupted during the storm. Although this plant was equipped with two 900 KW diesel generators, they were not sufficient to support all treatment processes. Besides the Coney Island and Rockaway plants, North River, Oakwood Beach, Port Richmond, 26th Ward, and Owls Head are the plants which experienced major flooding during Sandy. However, their different flood resilience attributes result in different resiliency values. The resilience of the Red Hook plant has increased significantly while the resilience of Coney Island has considerably decreased, which is more consistent with NYCDEP reports.

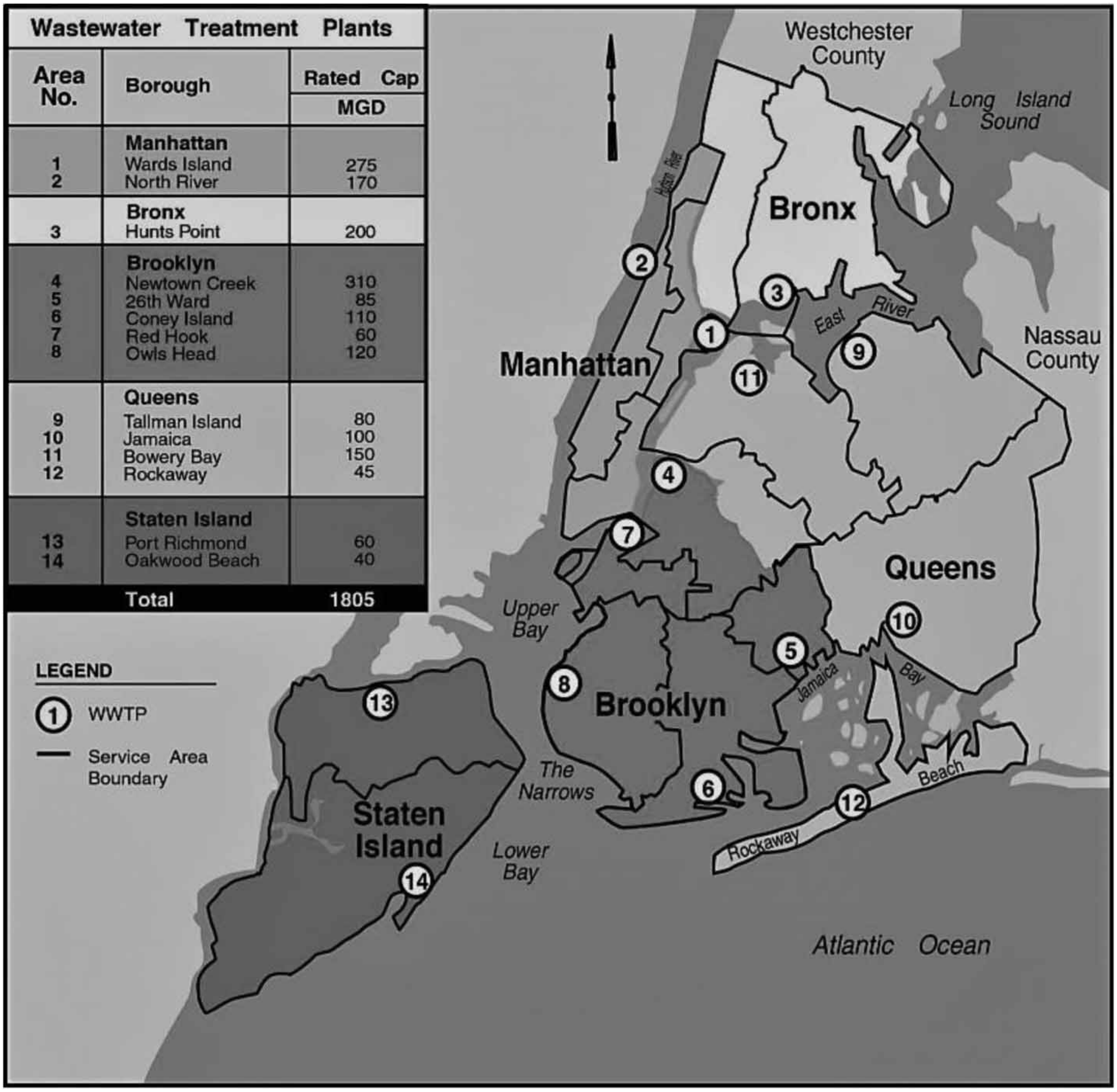

Figure 5 | NYC WWTPs service areas (adopted from Wilson *et al.* (2016)).

Considering that the implementation of resilience interventions needs investment in the systems, the resilience improvement of each system versus investment is illustrated in Figure 7. A logarithmic function $(aln(x) + b)$ is fitted to the behavior of each of these systems as their utilities of the received budgets. The derivative of these functions (a/x), which defines the marginal utility of NYCDEP (in CBT), is strictly decreasing and represents that the additional utility of NYCDEP reduces with spending more dollars on the plants and the axiom of law of diminishing marginal utility is satisfied.

The coefficient a indicates the rate of resilience improvement. Here, the Rockaway and Coney Island plants with coefficients of 2.92 and 2.48 have the highest resilience improvement rate, respectively. In other words, these plants have the highest potential for resilience improvement

Table 3 | Values of sub-criteria for WWTP facilities

WWTP	Bowery Bay	Hunts Point	Tallman Island	Wards Island	Newtown Creek	North River	Oakwood Beach	Port Richmond	Red Hook	26th Ward	Coney Island	Jamaica	Owls Head	Rockaway
Sub-criteria														
Ra_1	11.6	10.2	10.1	10.7	10.0	9.7	13.1	12.1	11.7	12.6	10.1	0.0	13.5	11.4
Ra_2	0	1	0	0	0	0	0	0	0	1	1	1	0	1
Ra_3	150	200	80	275	310	170	39.9	60	60	85	110	100	120	45
Ra_4	0	30	3	0	13	14	167	17	0	30	112	0	16	180
Ra_5	848	685	411	1,062	1,068	589	245	198	192	283	596	728	758	90
Ra_6	0	153.7	7.5	1.7	142.9	8.2	118.7	15.0	0.0	44.5	35.6	0	38.1	118.5
Ra_7	5,786	6,767	1,019	11,388	8,942	6,475	2,576	2,600	1,907	4,116	3,936	3,836	2,645	2,600
Ra_8	3,500	12,000	4,700	14,224	4,000	13,740	3,500	2,000	8,000	9,000	6,733	3,500	275	2,000
Ro_1	150	200	40	275	390	170	80	60	60	85	110	100	120	45
Ro_2	15.5	17.5	15.5	17.5	13.5	12.5	16.5	14.5	14.5	13.5	15.5	13.5	14.5	14.5
Ro_3	5	7	7	6	4	6	5	4	6	5	3	0	4	7
Ro_4	64	45	66	98	92	66	85	55	72	78	73	99	71	62
Ro_5	0	0.2	1.6	1.4	0.9	0.3	2	1.9	0	1.1	2.1	0.9	1.4	1.9
Ro_6	112.6	201.4	45.2	8.7	28.8	94.1	21.0	54.8	67.4	82.4	84.9	1.7	48.4	49.3
Ro_7	0.9	0.0	0.0	0.0	0.0	0.3	2.4	0.0	0.0	0.4	2.5	0.0	0.0	2.9
Ro_8	0	0	0	0	0	1	1	0	0	0	1	0	0	1
Rs_1	81	108	71	118	88	109	59	46	55	93	69	66	68	41
Rs_2	1	1	1	1	0	0	1	0	1	1	0	0	0	0
Rs_3	1.7	10.1	3.0	27.3	1.0	26.0	8.3	5.8	1.3	9.7	22.6	2.2	13.8	13.1
Rd_1	1	0	1	0	1	0	1	1	1	1	1	0	1	1
Rd_2	6.7	3.9	9.3	3.9	4.6	5.0	9.8	9.8	4.6	10.3	9.0	6.6	7.9	9.0
Rd_3	6.4	0	2.2	14.4	0	16.3	1.5	9.2	4.3	0	0	1.6	2.0	53.5
Rd_4	2	6	3	5	2	5	3	1	4	3	5	2	4	2

Table 4 | The proposed coefficients' values of financial constraints

Resilience intervention	Maximum percentage of total budget	Value (%)
Increasing treatment capacity	α	35
Water proofing, sand bagging, and elevating critical equipment	β	60
Increasing technical workers	γ	10
Increasing on-site storage	λ	35

and the highest investment priorities. Interestingly, these plants are those which have the least amount of resilience among others. In contrast, the Wards Island plant with the coefficient of 0.39 has the lowest potential to improve resilience. In comparison to the results of Karamouz *et al.* (2018), the priority of the plants in terms of investment is relatively stable; however, the resilient improvement rate of the plants has changed.

Because of the uncertainties previously discussed, the utility of the plants or the improvement of their resilience cannot be represented merely in the form of a fixed curve (solid lines in Figure 7). Rather, a set of curves could offer the utility of the systems in the process of distributing flood resilience resources. These curves are based on statistical distribution of random sub-criteria (Table 2A). The upper and the lower envelope of these curves are illustrated for each of these plants. These envelopes represent the maximum and minimum resilience improvement for a given budget, respectively. Accordingly, the plants can be classified into three categories. For some of the plants the fixed curve is very close to the lower envelope. The fixed utility of the Wards Island plant, for example, is almost touching the lower envelope. For some other plants, such as Hunts Point, the fixed curve is very close to the upper envelope. The fixed curve of others, such as Oakwood Beach, is almost in the middle of the upper and the lower envelope. The gap between the upper and the lower envelope of a plant implies the uncertainty of allocation to that plant. For example, due to the wide gap between the upper and the lower envelope of Wards Island, the financial allocation to this plant is expected to be more uncertain.

In the following subsections, the fixed curve of the systems is used for deterministic financial allocation and the set of curves defined between the upper and the lower envelope of each system is utilized for stochastic allocation among them.

DETERMINISTIC FINANCIAL ALLOCATION

Assuming that the resilience improvement of the plants is deterministic, the \$187 million budget of NYCDEP is distributed among them according to Figure 8. This budget is allocated to the plants based on maximizing overall resilience (MOR) and Nash product (MNP). For some of the plants, such as 26th Ward and Coney Island, the allocation based on MOR is very close to the NYCDEP's recommended values. The Coney Island plant, for example, receives \$15.5 million from the NYCDEP compared to \$15.3 million based on MOR. The Wards Island plant, which has the lowest potential for resilience enhancement,

Table 5 | Weights of criteria and sub-criteria

Rapidity (Ra)		Robustness (Ro)		Resourcefulness (Rs)		Redundancy (Rd)	
$W_{Ra} = 0.395$		$W_{Ro} = 0.349$		$W_{Rs} = 0.104$		$W_{Rd} = 0.151$	
Ra_1	0.064	Ro_1	0.035	Rs_1	0.028	Rd_1	0.049
Ra_2	0.015	Ro_2	0.064	Rs_2	0.056	Rd_2	0.035
Ra_3	0.064	Ro_3	0.022	Rs_3	0.021	Rd_3	0.028
Ra_4	0.057	Ro_4	0.029			Rd_4	0.04
Ra_5	0.064	Ro_5	0.069				
Ra_6	0.069	Ro_6	0.068				
Ra_7	0.032	Ro_7	0.032				
Ra_8	0.032	Ro_8	0.032				

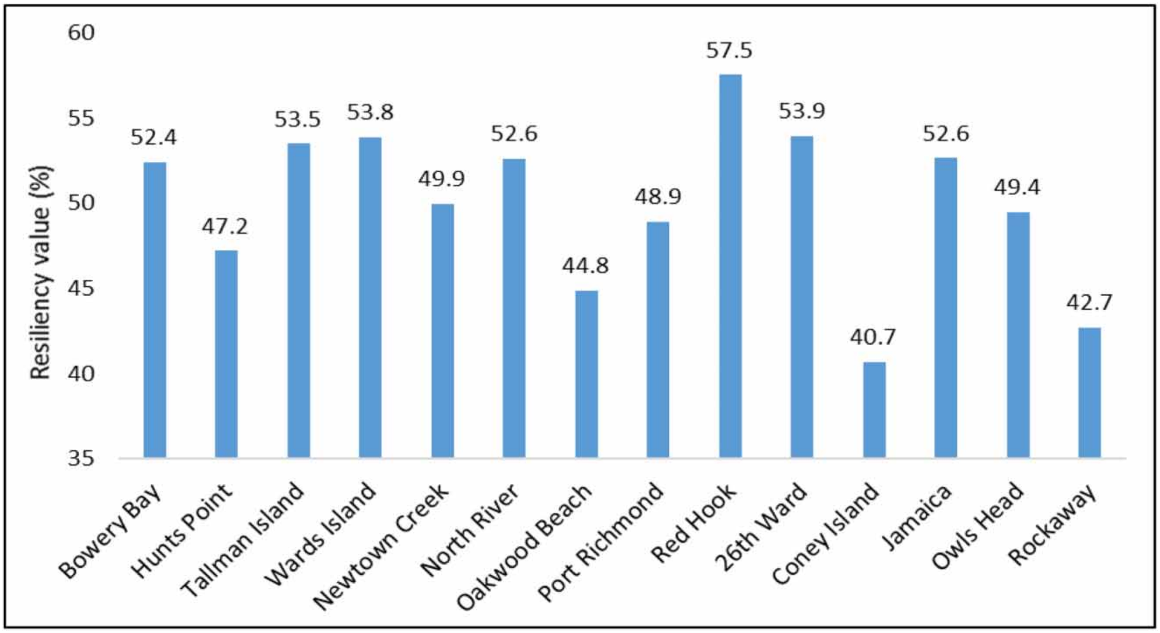

Figure 6 | Resilience values of WWTPs based on MCDM.

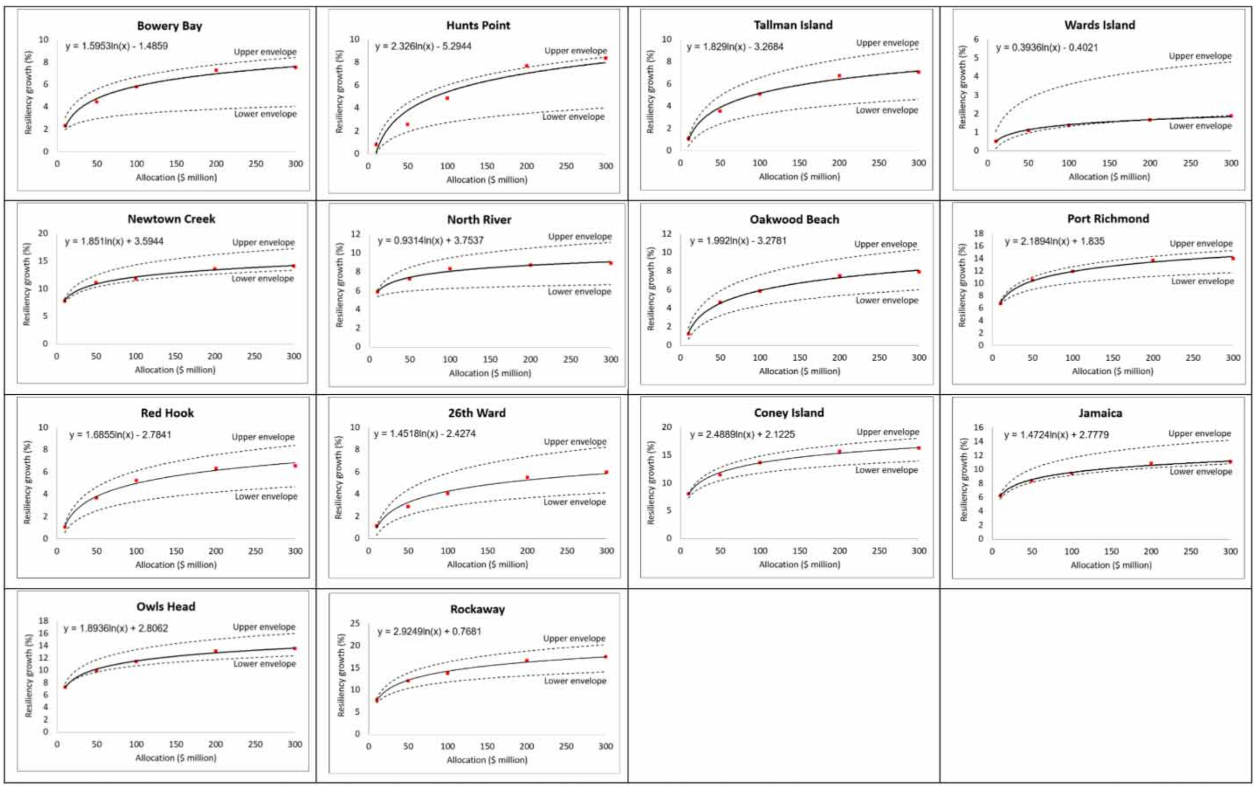

Figure 7 | Financial allocation-resilience improvement curve (utility function) of WWTPs (dashed lines are the upper and lower envelopes representing maximum and minimum resilience improvement for a given budget, respectively).

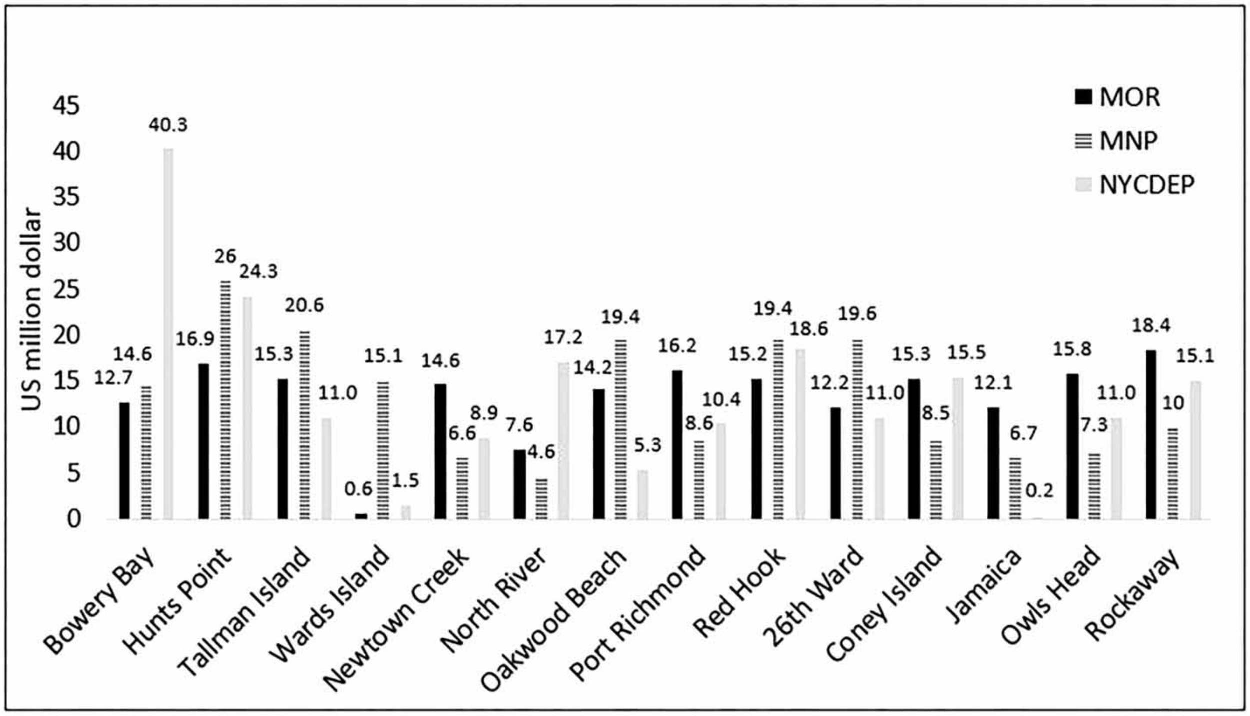

Figure 8 | Deterministic financial allocation based on maximizing overall resilience (MOR) and Nash product (MNP).

almost receives no allocation while the smaller plant, Rockaway, with low resilience and high potential of improvement, receives the maximum allocation and resilience benefit. This is because many factors, including the depth of flood, contribute to the utility of each plant and the Wards Island plant is less vulnerable to coastal flooding compared to the Rockaway plant which suffers direct surge impact. For some other plants, such as Hunts Point, the allocation based on MNP is closer to the recommended allocation of NYCDEP. Based on the MNP approach, the Hunts Point plant receives the highest allocation of $26 million which is close to the NYCDEP's $24.2 million while the North River, with $4.6 million, receives the lowest.

Figure 9 shows the resilience improvement of the plants associated with the financial allocation performed above. The Wards Island plant is not improved based on MOR. The maximum resilience improvement is also related to the Rockaway plant with 9.4%. Interestingly, these two plants had received the highest and the lowest allocations, respectively. Similar to MOR, the lowest resilience improvement based on MNP is associated with the Wards Island plant, but in this case the resilience of this plant is slightly improved. Rockaway is still the most improved plant; however, this improvement has decreased to 7.5%. In contrast to MOR, the maximum and minimum resilience improvement is not related to the plants that had received the highest and lowest allocations (Hunts Point and North River in Figure 8), respectively. In general, plants with lower resilience improvement than the MOR average resilience improvement of 4.6%, including Bowery Bay, Hunts Point, Tallman Island, Wards Island, Oakwood Beach, and 26th Ward, have higher resilience improvement in MNP compared to MOR.

Among all the plants, the budgets allocated to the Wards Island plant are significantly different. This plant almost receives no fund based on MOR but receives a significant allocation of $15.1 million based on MNP. This difference arises because in MOR as long as the overall resilience improvement of systems is increased, any budget distribution is acceptable even if a system receives nothing from the budget. However, MNP has a strong tendency to distribute the budget to minimize dissatisfaction. In other words, this allocation scheme tends to increase the

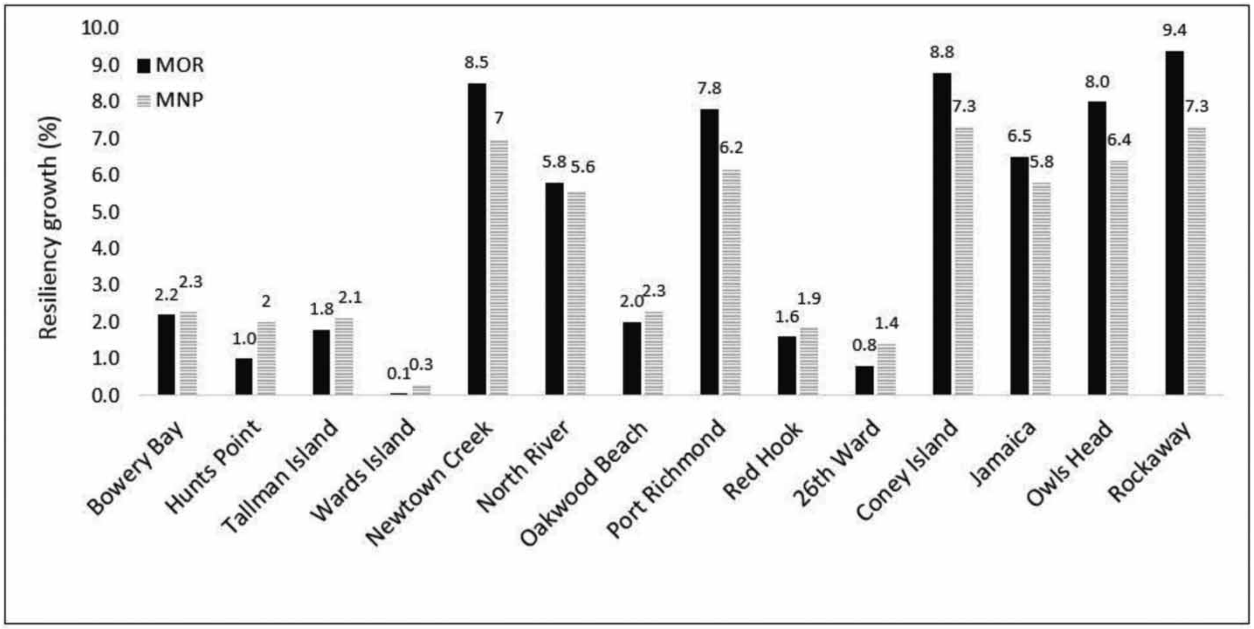

Figure 9 | Resilience improvement of the plants associated with the deterministic allocation.

resilience improvement of the system with minimal improvement. Since the Wards Island plant has the lowest improvement of resilience among other plants (see Figure 9), the maximum difference of the two allocation schemes belongs to this plant.

Depending on organizational strategic goals, any of these allocations can be acceptable. The result of consumer behavior theory (CBT) is identical to those of MOR and for the sake of brevity we skipped presenting CBT results. The difference between MNP and CBT is that in MNP it is assumed that each plant has its own utility in the bargaining process. In other words, 14 players participate in the bargaining process in order to maximize their benefits, while in CBT, utilities are defined for one player (NYCDEP agency) that wants to maximize its benefit from financial allocation to its facilities.

UNCERTAINTY BASED ALLOCATION

Stochastic uncertainty

Here, distributing the budget of $187 million among the plants is investigated in an uncertain environment. The probability distribution of allocation for each plant based on MOR and MNP is illustrated in Figure 10. Using Monte Carlo sampling from random sub-criteria's distributions (see Table 2A) and examining null hypothesis in the Chi Square test, it was observed that a random sample of allocation to each plant accepts the null hypothesis and therefore normal distribution was fitted to random allocations to each plant. The standard deviation (SD) of distribution for the plants shows the degree of allocation uncertainty. In this figure, the cumulative distribution represents the probability that the optimal allocation will take an amount less than or equal to a certain amount of budget. On this basis, in the case of Bowery Bay, the probability that the optimal allocation is less than or equal to $10 million is almost 70 and 40% based on MOR and MNP, respectively. A similar observation can be extended to other plants.

According to MOR, investment in the Hunts Point and 26th Ward plants are associated with the highest risk and uncertainty. Here the risk of investment means getting lower resilience improvement than expected for a given allocation (see Figure 10). Therefore, optimal investment in these plants is associated with more risk. On this basis, the Bowery Bay plant has the lowest investment risk.

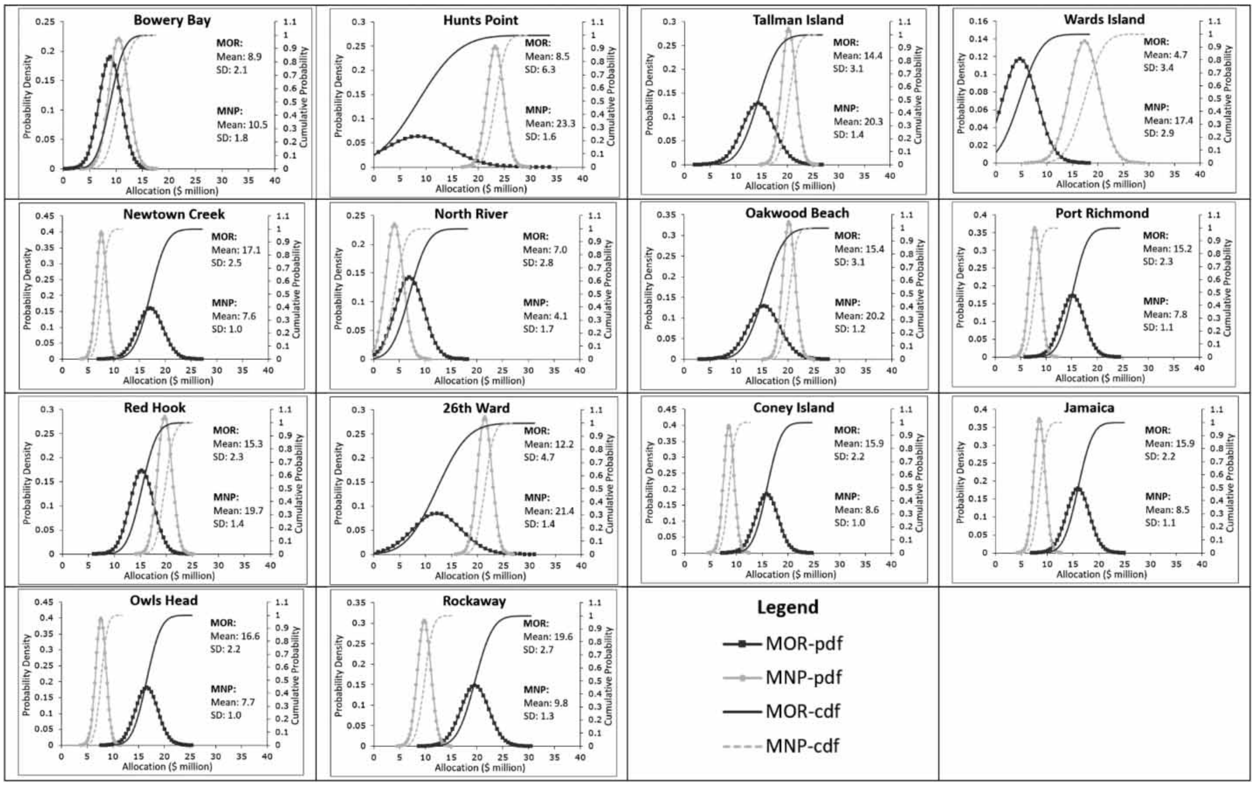

Figure 10 | Financial allocation distribution of 14 WWTPs in NYC based on maximizing the Nash product (MNP).

In allocation based on MNP, the standard deviation of allocation for the plants becomes smaller. In this case, the SD ranges are between 1.0 and 2.9 and the Wards Island plant has the highest SD or allocation uncertainty. The 95% confidence interval of allocation for each of these plants is illustrated in Figure 11 based on MOR and MNP.

The difference at the 95% confidence intervals indicates that the use of different allocation schemes influences the degree of uncertainty in the allocation process.

Regarding different investment risks of allocations, it is important to know which allocation option is associated with the lowest risk. For this purpose, the financial allocation with the lowest investment risk is determined in Figure 12 based on maximizing the expected value of MOR and MNP.

Based on maximizing the expected value of overall resilience, the Rockaway plant still receives the highest share of the total budget with $19.2 million, and the Wards Island plant receives almost no fund similar to the deterministic allocation. In this case, the maximum change compared to the deterministic allocation is related to the Hunts Point plant, which is a $4.1 million decrease.

By maximizing the expected value of the Nash product, the highest budget is received by the Hunts Point plant with $23.1 million and the lowest by the North River with $4.1 million.

Therefore, the plants that received the highest and the lowest funding from the deterministic allocation do not change. In this case, the most notable allocation change is for the Bowery Bay plant but with a $3.8 million decrease compared to the deterministic allocation based on MNP.

Subjective uncertainty

Due to different analysis and expert perspectives on allocation to each infrastructure, assessing the uncertainty of allocation to each plant can help managers of these facilities make the right decisions and manage their finances. Therefore different beliefs are combined for each plant, and the range of belief, disbelief, and uncertainty are measured

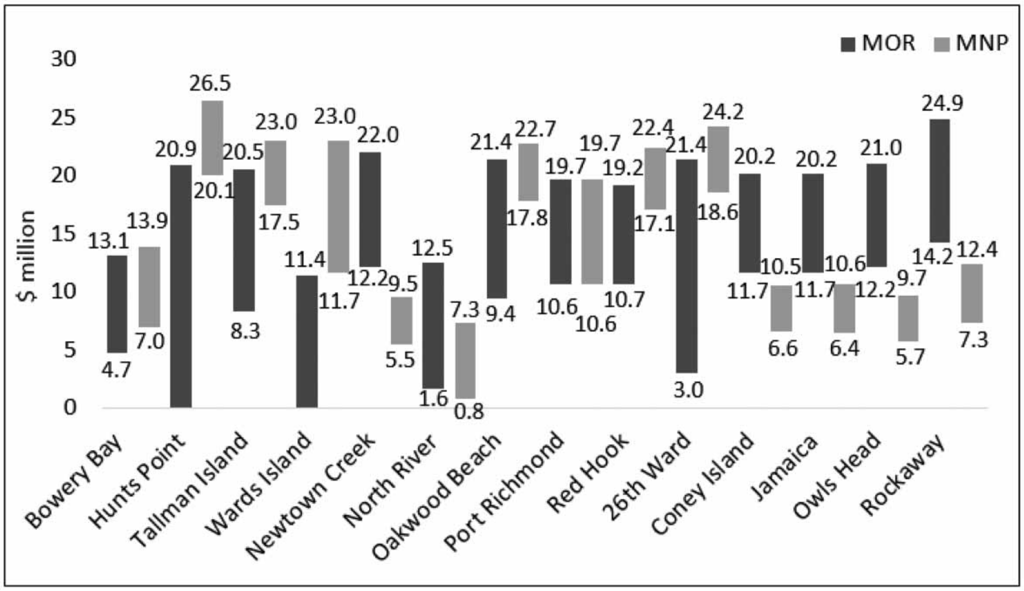

Figure 11 | 95% confidence interval of allocation based on maximizing overall resilience (MOR) and Nash product (MNP).

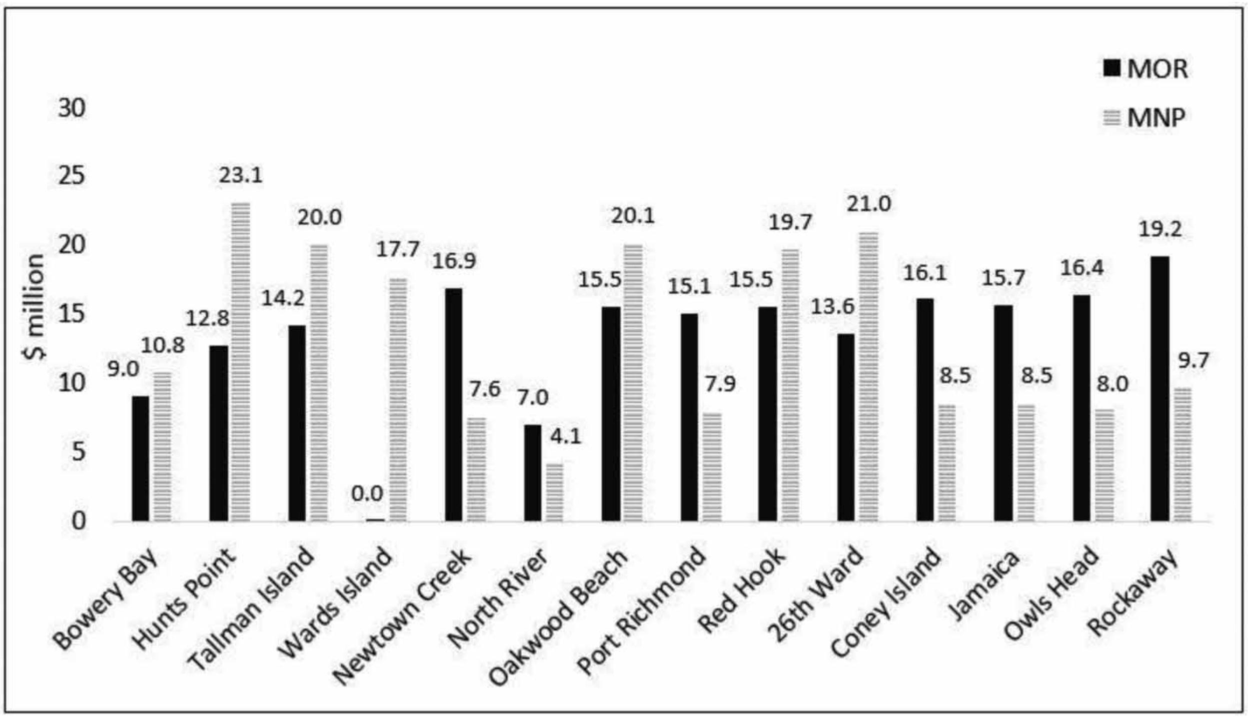

Figure 12 | Stochastic allocation based on maximizing the expected value of overall resilience (MOR) increase and Nash product (MNP).

over each proposition of allocation. An expert based on the resilience analysis of this study and another based on the building and infrastructure-level vulnerability assessment performed by the NYCDEP, arrive at the degrees of belief in each proposition. The belief mass distribution of the latter expert based on the interval of NYCDEP's allocation is assumed trapezoidal. Accordingly, the mass of belief in the allocation interval of this agency is 50%, and in the adjacent intervals of 25% each (Figure 13(a)). These values for the first allocation interval are 67 and 33%, respectively (Figure 13(b)).

The belief and disbelief intervals and the range of uncertainty over each proposition of allocation are illustrated in Figure 14 based on the evidence theory and Dempster–Shafer rule of combination (Equations (10)–(16)). The black bars in this figure show the degree of inconsistency between the two expert beliefs for each plant. According to these bar charts, the most inconsistency corresponds to the Bowery Bay, Hunts Point, and Jamaica plants with almost 68%.

The Dempster–Shafer rule of combination is inappropriate for inference from highly inconsistent beliefs because it does not take into account the inconsistency between beliefs (Agarwal *et al.* 2004). Therefore, no inference is made about the allocation to the Bowery Bay, Hunts Point, and Jamaica plants. For the other plants, the inconsistency between the beliefs is far lower. In the cases of the Tallman Island, Wards Island, Port Richmond and Owls Head plants, this is less than 5%.

In the cases of the Newtown Creek, North River and Oakwood Beach plants, evidence rejects the allocation types of low and high, and the degrees of plausibility and belief are almost zero. While the evidence almost entirely supports the allocation type of medium, and these plants are placed in the category of plants for which the allocation type can be defined as medium.

The inconsistency between the expert beliefs for the Tallman Island plant is negligible and about 3%. The low-type of allocation to this plant is refuted by nearly all the evidence and the degree of belief in the high-type of allocation is negligible. Therefore, the allocation to this plant can be interpreted as medium.

In the case of the Wards Island plant, the inconsistency between the expert beliefs is almost 4%. Given the high-type allocation is refuted by the evidence and the degree of belief in the allocation type of medium is negligible, the allocation to the plant could be considered the type of low.

The conflict between experts' beliefs for the Port Richmond plant is only 1%. The evidence denies the allocation type of low for this plant. Furthermore, the agree of belief in the allocation type of high is negligible. Hence, the allocation type of medium could be considered for this plant.

The inconsistency between the expert beliefs for the Red Hook plant is almost 12%. The evidence refutes the allocation type of low for this plant. The degree of belief in the allocation types of medium and high are 53 and 17%, respectively. Given that most evidence supports the medium-type of allocation and the upper limit of belief in the allocation type of high is less than 50% (Pl = 45%), the allocation to the plant could be defined as medium.

In the case of the 26th Wards Plant, the conflict between the expert beliefs is 8%. The degrees of belief in the high and low-type of allocations to this plant are insignificant. The degree of belief in the medium-type of allocation is 84%,

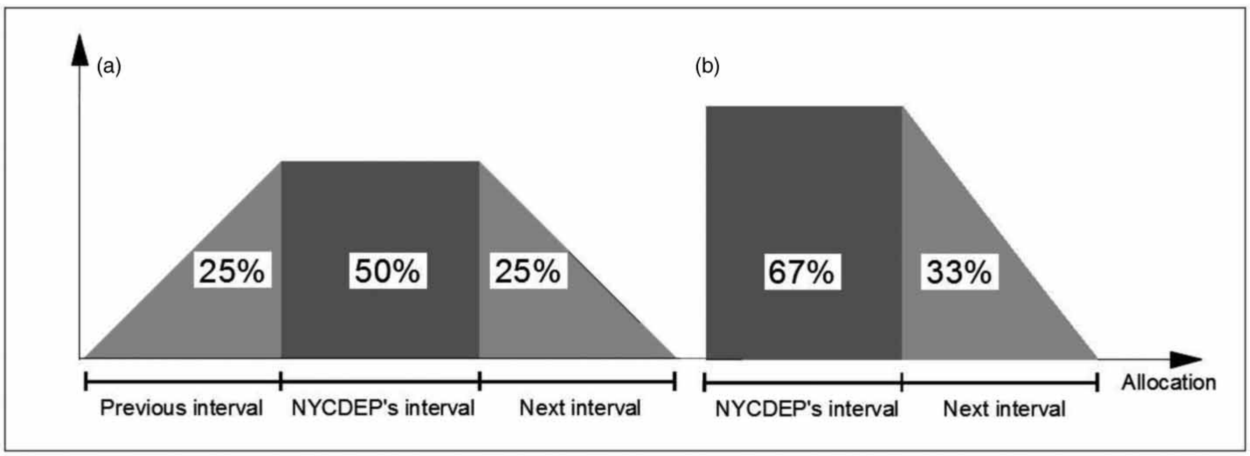

Figure 13 | Belief mass distribution of the second expert assumed based on the NYCDEP's interval of allocation: (a) for the intermediate interval and (b) for the first interval.

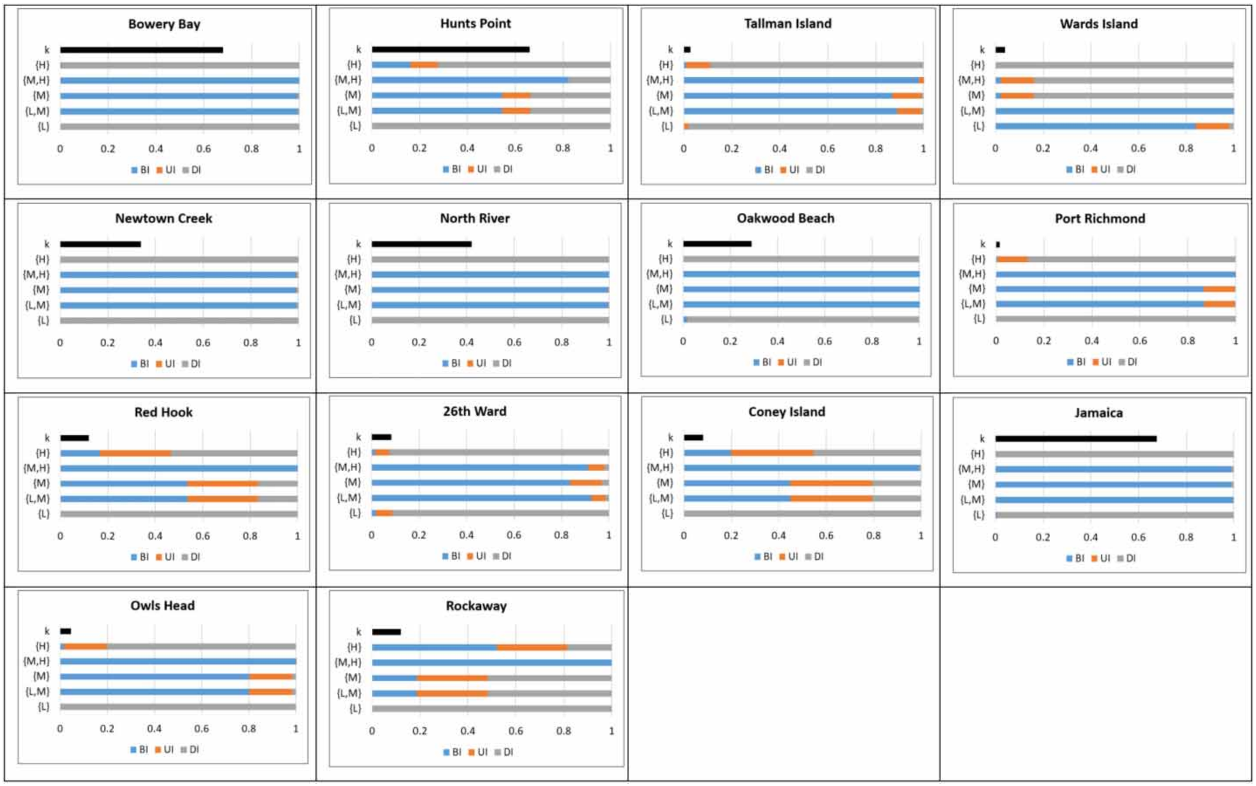

Figure 14 | Belief (BI), uncertainty (UI) and disbelief intervals (DI) of financial allocation (k: inconsistency between expert beliefs, {H}: high, {M,H}: either medium or high, {M}: medium, {L,M}: either medium or low, and {L}: low allocation).

which can increase to 97% with uncertainty (Pl = 97%). Therefore, the allocation to this plant could be interpreted as medium-type.

The inconsistency between the expert beliefs for the Coney Island plant is almost 8%. The evidence denies the low-type of allocation to this plant. The degrees of belief in the allocation types of medium and high are 44 and 20%, respectively. the plausible interval of these two types of allocations are 80 and 55%, which are greater than 50%. Therefore, the allocation type for the plant could be defined as either medium or high.

In the case of the Owls Head plant, there is only 5% inconsistency between the beliefs of the experts. The evidence refutes the low-type of allocation to this plant. The degree of belief in the high-type of allocation is also insignificant. The allocation to this plant, therefore, could be considered as medium-type.

Finally, the inconsistency between the expert beliefs for the Rockaway plant is almost 12%. The evidence denies the low-type of allocation to this plant. The degree of belief in the high-type of allocation is 52%. Although the belief's degree in the medium-type of allocation is 19%, the plausibility of this belief is less than 50%. Therefore, the allocation to this plant could be considered as high-type. The type of allocation to each plant is illustrated in Table 6.

SUMMARY AND CONCLUSION

Wastewater treatment plants are vulnerable infrastructures against coastal storms. The failure of these facilities and the interruption of their recovery will result in irreparable environmental consequences. Improving the flood resilience of these systems is essential and requires considerable funds. However, due to budget limitations it is inevitable to optimize the allocation of financial resources among them. The optimal allocation among these systems needs an assessment of their performance indicators such as resiliency. For such an assessment the characteristics of these systems in terms of hydrological, environmental, economic, technical and operational aspects have been taken into account. Also, given the potential power outages during floods and the urgent need of these systems for power, the dependence of these facilities on energy has been included in the assessment of their flood resilience. All flood resilience attributes of these systems are categorized in terms of rapidity, robustness, resourcefulness and redundancy (four Rs).

The allocation of resources among the facilities of an agency is typically performed based on non-technical negotiations and default allocation routine. This study has provided a robust metric based on expert views for reinforcing the resource allocation mechanism of water/wastewater agencies. Using multiple expert views, the relative importance of four Rs has been determined. Since more weights were assigned to rapidity and robustness, the importance of these criteria is more dominating to evaluate the resilience of these facilities. Assessing the resilience of NYC's wastewater treatment plants, with regard to interdependencies, shows better agreement with NYCDEP reports after Superstorm Sandy. In other words, the flood resilience of the plants such as Coney Island that were seriously damaged in Superstorm Sandy have been far less than the plants that were not exposed to major damage. Considering the dependence of sewage treatment facilities on energy and the related flood resilience attributes such as the number of backup generators and their capacity, a better comparison of their resiliency has been made in Figure 6.

Taking into account different resilience interventions and related costs, the resilience improvement of each plant was determined for a wide range of allocations and is assumed as their utilities of the allocations. Assuming that these utilities are deterministic, the allocation of resources has been implemented based on maximizing overall resilience (MOR), Nash product (MNP), and consumer behavior theory (CBT). Allocation based on consumer behavior theory confirms the result of MOR. The Rockaway and Wards Island plants have the highest and the lowest resilience improvement, respectively in both MOR and MNP. Based on MOR, these two plants have also received the highest and the lowest allocations, respectively. The Wards Island plant has received insignificant allocation in MOR. However, in MNP, the highest and the lowest allocations are not attributed to these plants. In contrast to MOR, Wards Island has received significant allocation in MNP. In other words, the resilience improvement of Wards Island with minimal improvement increases in MNP, which demonstrates that this allocation scheme tends to increase the resilience improvement of the system with minimal improvement and to distribute funds more uniformly.

Table 6 | Type of allocation to WWTPs

WWTP	Allocation type	WWTP	Allocation type
Bowery Bay	N/A[a]	Port Richmond	Medium
Hunts Point	N/A	Red Hook	Medium
Tallman Island	Medium	26th Ward	Medium
Wards Island	Low	Coney Island	Either medium or high
Newtown Creek	Medium	Jamaica	N/A
North River	Medium	Owls Head	Medium
Oakwood Beach	Medium	Rockaway	High

[a]Not applicable due to high inconsistence between expert beliefs.

Due to the uncertainties in flood characteristics and some of the resilience attributes of these facilities, they were considered in the utility of each plant. Accordingly, allocations were made among the plants using MOR and MNP. Based on the standard deviation and the 95% confidence interval of allocations, it was demonstrated that the level of uncertainty is much lower in MNP compared to MOR. Therefore, the level of uncertainty is influenced by how resources are distributed.

Considering the different views of experts in the allocation process, the subjective uncertainties of each plant have been examined by the theory of evidence. Accordingly, the degree of inconsistency between experts' beliefs was measured and the belief, plausibility and uncertainty were estimated for each proposition of allocation. The results showed that the inconsistency is high in the cases of Bowery Bay, Hunts Point and Jamaica plants and is far less for the other plants. For plants where the degree of inconsistency is high on the allocation intervals, the Dumpster rule of combination cannot be applied and the uncertainty intervals of these plants cannot be determined. Subsequently, for other plants, according to the degree of belief in each proposition, the type of allocation has been determined. The results show that the medium-type of allocation could be considered for most of the plants.

The result of this study demonstrates the significant value of resilience based funds considering interdependencies and uncertainties related to different views of entities and analyses. The methodology of the paper is applicable to other geographic settings in coastal areas. One limitation of this framework is the sensitivity of its results to model input subjective values. A sensitivity analysis performed by Karamouz *et al.* (2018) on the input of the model indicates that plants with high investment potential are more sensitive to the model's subjective inputs. Global sensitivity analysis on the dependence of the results on the model inputs is suggested as it is outside the scope of this study.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this paper is available online at https://dx.doi.org/10.2166/hydro.2020.145.

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