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Fuzzy credibility-constrained quadratic optimization for booster chlorination of the water distribution system under uncertainty

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ABSTRACT

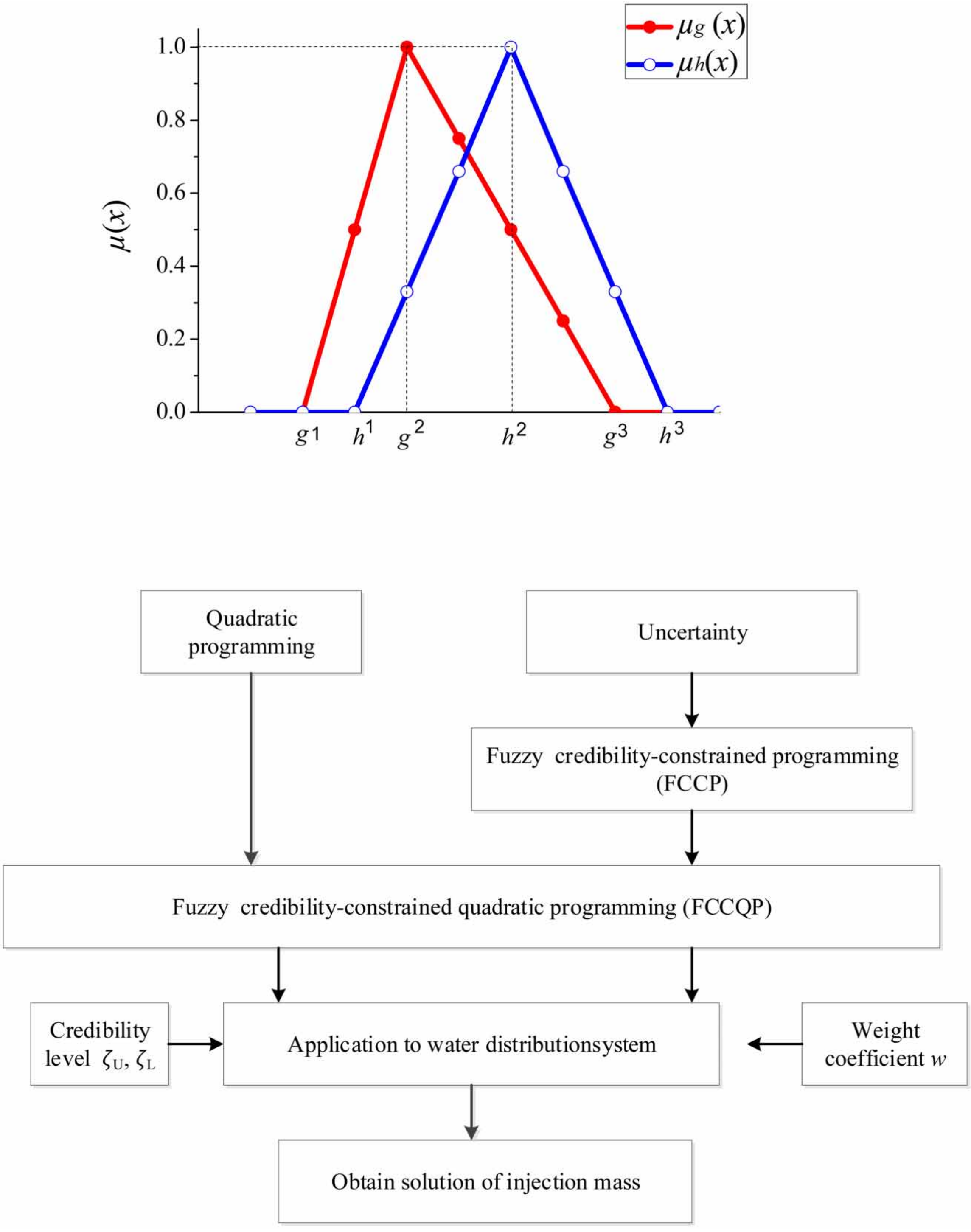
To keep chlorine concentration at acceptable levels, chlorine is usually injected into the water distribution system (WDS). To protect the health of human beings, the chlorine concentration at consumers' nodes should be kept at appropriate levels. However, these levels are difficult to determine due to the presence of fuzzy uncertainties. To deal with fuzziness at both sides of the constraints in the optimization model of booster chlorination, we propose a fuzzy credibility-constrained quadratic programming (FCCQP) model with a consideration of credibility levels and weight coefficients. The proposed model is applied to two WDSs to obtain the booster cost under uncertain conditions. The results indicate that the booster cost increases with the confidence level for lower chlorine concentration ζ_L. In addition, the booster cost decreases with the weight coefficient w. The booster cost function curves along with the variation of weight coefficients are concave and convex for scenario 1 and scenario 2, respectively. These results can help managers to make informed decisions on disinfection injection under conditions of fuzzy uncertainties.

Key words: booster cost, confidence level, optimization, water distribution system, weight coefficients

HIGHLIGHTS

- The study deals with fuzziness at both sides of the constraints.
- The study integrates the generalized fuzzy credibility chance-constrained programming and quadratic programming.
- The study obtains booster cost under various credibility levels and weight coefficients.
- The study analyzes the proportion of the operation cost.
- The study compares the booster costs of various cases.

GRAPHICAL ABSTRACT



1. INTRODUCTION

Generally, chlorine is injected into the water distribution system (WDS) at treatment plants as disinfectants. However, chlorine decay along the WDS due to its reaction with organic matter in bulk water and pipe wall, which leads to residual chlorine concentration at nodes far from water sources. On the other hand, the chlorine concentration at nodes near water sources is usually higher, which leads to the formation of disinfectant byproducts (Boccelli *et al.* 2003; Basupi & Nono 2019). As such, the chlorine residual concentration in the WDS should be within acceptable limits (Köker & Altan-Sakarya 2015). Wang *et al.* (2019) investigated the risks brought about by chlorination, and concluded that chlorine levels should be reduced to decrease the risks of both cancer and non-cancer conditions. Many models were proposed to optimize the numbers, locations and injection rates of booster. Tryby *et al.* (2002) proposed a mixed-integer linear programming model to optimize locations and scheduling of booster chlorination (Al-Zahrani 2016). Prasad *et al.* (2004) proposed a multi-objective genetic algorithm (MOGA) model to minimize the total disinfectant dose and maximize the volume of water supplied with acceptable residual chlorine (Tryby *et al.* 2002). Munavalli & Kumar (2003) used binary strings to code the chlorine dosages to determine chlorine injection rates at defined booster locations with maximum and minimum constraints of chlorine concentration. Propato & Uber (2004) proposed a linear least-squares (LLS) model to optimize disinfectant injection rates to minimize the sum of the squared deviations of residual concentrations from a desired target. Ostfeld & Salomons (2006) proposed a conjunctive optimal schedule to minimize the cost of pumping and chlorine booster design and operation and maximize the injection of the chlorine dose. However, chlorine concentration in the WDS is related to many factors such as water demand, pipe roughness, chlorine bulk decay coefficient, and chlorine wall decay coefficient, which leads to cognitive uncertainty and is difficult to obtain (Xu & Qin 2014). In addition, the maximum and minimum bounds of residual chlorine concentration in the WDS vary with regulations imposed in different countries and regions, which also have uncertainties.

With a consideration of uncertainties in nodal demands and pipe roughness coefficients, a chance-constrained programming (CCP) model was proposed to minimize the cost of the WDS (Babayan *et al.* 2005), which can deal with the randomness on the right-hand side of the constraints (Zhao *et al.* 2016). A fuzzy framework was also introduced to deal with the uncertainty in the WDS. Xu and Qin (2014) integrated fuzzy programming and decision analysis to deal with the fuzzy objective function and both sides of constraints in the management framework of the WDS. Geem (2015) proposed a fuzzy-based velocity reliability index in the optimization of WDS design. Moosavian and Lence (2018) approximated the fuzziness of nodal pressures under fuzzy nodal demands and pipe roughness coefficients. Xu & Qin (2014) integrated the decision analysis and fuzzy programming to help analyze the tradeoffs between the minimization of the operation cost and maximization of reliability. A fuzzy chance-constrained programming (FCCP) model was proposed to solve the scheduling of booster disinfection to reflect the ambiguity in the constraints, which can deal with the fuzzy uncertainty on the right-hand side of the constraints (Wang 2021). Additionally, Wang proposed an inexact left-hand side chance-constrained programming (ILCCP) model to deal with the interval fuzziness in the objective function and left-hand side of the constraints (Wang & Zhu 2021a). However, the fuzziness exists not only on the left-hand-side of the constraints but also on the right-hand-side of the constraints. In addition, booster cost consists of not only the operation cost that is closely connected with the injection mass but also the construction cost. Against this background, in this paper, a fuzzy credibility–constrained quadratic optimization (FCCQP) model is proposed to bridge this gap. This model integrates both fuzzy credibility-constrained programming (FCCP) and quadratic programming (QP) into the optimization framework. This model can compare the results obtained under various credibility levels and weight coefficients.

In Section 2 of this paper, a FCCQP model is introduced to minimize the booster cost of the WDS. The FCCQP model is transformed into linearization form and applied to two WDSs in Section 3. The results obtained through the FCCQP model are analyzed and discussed in Section 4. In Section 5, the conclusion is drawn.

2. METHODOLOGY

2.1. Quadratic programming

The QP model can deal with nonlinearities in programming effectively, which is expressed by Equation (1) as follows:

$$Min f = \sum_{j=1}^{n} [c_j + d_j(x_j)^2], \quad j = 1, 2, \ldots, n \tag{1a}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1, 2, \ldots, m \tag{1b}$$

$$x_j \ge 0 \tag{1c}$$

where $a_{ij}, b_i, c_j,$ and d_j are coefficients in objective functions and constraints, and x_j is the decision variable. However, QP cannot deal with the uncertain fuzzy information (Guo *et al.* 2015).

2.2. Fuzzy credibility-constrained quadratic programming

2.2.1. Fuzzy credibility-constrained programming

The FCCP model can be expressed by Equation (2) as follows:

$$Min f = \sum_{j=1}^{n} c_j x_j \tag{2a}$$

subject to

$$Cr\left\{\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \leq \widetilde{b_U}\right\} \geq \zeta_U, \quad i = 1, 2, \ldots, m \tag{2b}$$

$$Cr\left\{\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \geq \widetilde{b_L}\right\} \geq \zeta_L, \quad i = 1, 2, \ldots, m \tag{2c}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{2d}$$

The membership function of variable x to a fuzzy set f expressed by triangular distribution (f^1, f^2, f^3) is termed $\mu(x)$, which is expressed by Equation (3) as follows:

$$\mu(x) = \begin{cases} \dfrac{x - f^1}{f^2 - f^1}, & f^1 \leq x < f^2 \\ \dfrac{x - f^3}{f^2 - f^3}, & f^2 \leq x < f^3 \\ 0, & \text{others} \end{cases} \tag{3}$$

For two fuzzy variables $\tilde{g}$ and $\tilde{h}$, the fuzzy events of $\tilde{g} \leq \tilde{h}$ and $\tilde{g} \geq \tilde{h}$ may have occurred (shown in Figure 1).

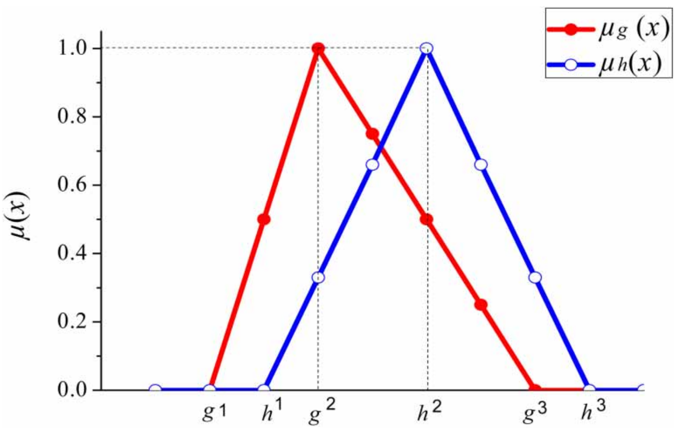


Figure 1 | Fuzzy variables $\tilde{g}$ and $\tilde{h}$.

The possibility of fuzzy events $\tilde{g} \leq \tilde{h}$ and $\tilde{g} \geq \tilde{h}$ is termed $Pos(\tilde{g} \leq \tilde{h})$ and $Pos(\tilde{g} \geq \tilde{h})$, respectively, which are expressed by Equation (4) as follows:

$$Pos(\tilde{g} \leq \tilde{h}) = sup\{\min(\mu_{\tilde{g}}(x), \mu_{\tilde{h}}(y))|x, y \in \Re, x \leq y\} \tag{4a}$$

$$Pos(\tilde{g} \geq \tilde{h}) = sup\{\min(\mu_{\tilde{g}}(x), \mu_{\tilde{h}}(y))|x, y \in \Re, x \geq y\} \tag{4b}$$

In addition, the necessity of fuzzy events $\tilde{g} \leq \tilde{h}$ and $\tilde{g} \geq \tilde{h}$ is termed $Nec(\tilde{g} \leq \tilde{h})$ and $Nec(\tilde{g} \geq \tilde{h})$, respectively, which are expressed as follows:

$$Nec(\tilde{g} \leq \tilde{h}) = inf\{\max(\mu_{\tilde{g}}(x), 1 - \mu_{\tilde{h}}(y))|x, y \in \Re, x \leq y\} \tag{5a}$$

$$Nec(\tilde{g} \geq \tilde{h}) = inf\{\max(\mu_{\tilde{g}}(x), 1 - \mu_{\tilde{h}}(y))|x, y \in \Re, x \geq y\} \tag{5b}$$

By combining with the definition of fuzzy membership function, $Pos(\tilde{g} \leq \tilde{h})$ and $Pos(\tilde{g} \geq \tilde{h})$ can be obtained and expressed by Equation (6) as follows:

$$Pos(\tilde{g} \leq \tilde{h}) = \begin{cases} 1, \quad g^2 \leq h^2 \\ \dfrac{h^3 - g^1}{h^3 - h^2 + g^2 - g^1}, \quad g^2 > h^2, g^1 \leq h^3 \\ 0, \quad g^1 > h^3 \end{cases} \tag{6a}$$

$$Pos(\tilde{g} \geq \tilde{h}) = \begin{cases} 1, \quad g^2 \geq h^2 \\ \dfrac{g^3 - h^1}{g^3 - g^2 + h^2 - h^1}, \quad g^2 < h^2, g^3 \geq h^1 \\ 0, \quad g^3 < h^1 \end{cases} \tag{6b}$$

Similarly, by combining with the definition of the fuzzy membership function, $Nec(\tilde{g} \leq \tilde{h})$ and $Nec(\tilde{g} \geq \tilde{h})$ can be obtained and expressed by Equation (7) as follows:

$$Nec(\tilde{g} \leq \tilde{h}) = \begin{cases} 1, \quad g^3 \leq h^1 \\ \dfrac{h^2 - g^2}{g^3 - g^2 + h^2 - h^1}, \quad g^2 < h^2, g^3 > h^1 \\ 0, \quad g^2 \geq h^2 \end{cases} \tag{7a}$$

$$Nec(\tilde{g} \geq \tilde{h}) = \begin{cases} 1, \quad g^1 \geq h^3 \\ \dfrac{g^2 - h^2}{h^3 - h^2 + g^2 - g^1}, \quad g^2 > h^2, g^1 \leq h^3 \\ 0, \quad g^2 \leq h^2 \end{cases} \tag{7b}$$

With a consideration of the weights of possibility and necessity measures, the credibility measure can be expressed by Equation (8) as follows:

$$Cr(\tilde{g} \leq \tilde{h}) = w \times Pos(\tilde{g} \leq \tilde{h}) + (1 - w) \times Nec(\tilde{g} \leq \tilde{h})) \tag{8a}$$

$$Cr(\tilde{g} \geq \tilde{h}) = w \times Pos(\tilde{g} \geq \tilde{h}) + (1 - w) \times Nec(\tilde{g} \geq \tilde{h})) \tag{8b}$$

The weight coefficient is in the interval [0.1, 1.0]. By combining Equations (6) and (7), Equation (8) can be transformed into Equation (9), which is expressed as follows:

$$Cr(\tilde{g} \leq \tilde{h}) = \begin{cases} 1, & g^3 \leq h^1 \\ \dfrac{w(g^3 - h^1) + h^2 - g^2}{g^3 - g^2 + h^2 - h^1}, & g^2 \leq h^2, g^3 > h^1 \\ \dfrac{w(h^3 - g^1)}{h^3 - h^2 + g^2 - g^1}, & g^2 > h^2, g^1 < h^3 \\ 0, & g^1 \geq h^3 \end{cases} \tag{9a}$$

$$Cr(\tilde{g} \geq \tilde{h}) = \begin{cases} 1, & g^1 \geq h^3 \\ \dfrac{w(h^3 - g^1) + g^2 - h^2}{h^3 - h^2 + g^2 - g^1}, & g^2 > h^2, g^1 \leq h^3 \\ \dfrac{w(g^3 - h^1)}{g^3 - g^2 + h^2 - h^1}, & g^2 \leq h^2, g^3 > h^1 \\ 0, & g^3 \leq h^1 \end{cases} \tag{9b}$$

when the weight coefficient w is equal to 1.0, Equations (8a) and (8b) are transformed to Equations (6a) and (6b). When the weight coefficient w is equal to 0.0, Equations (8a) and (8b) are transformed to Equations (7a) and (7b). When the weight coefficient w is equal to 0.5, Equations (8a) and (8b) are transformed to Equations (10a) and (10b), which are expressed as follows:

$$Cr(\tilde{g} \leq \tilde{h}) = \begin{cases} 1, & g^3 \leq h^1 \\ \dfrac{g^3 - 2g^2 + 2h^2 - h^1}{2(g^3 - g^2 + h^2 - h^1)}, & g^2 \leq h^2, g^3 > h^1 \\ \dfrac{h^3 - g^1}{2(h^3 - h^2 + g^2 - g^1)}, & g^2 > h^2, g^1 < h^3 \\ 0, & g^1 \geq h^3 \end{cases} \tag{10a}$$

$$Cr(\tilde{g} \geq \tilde{h}) = \begin{cases} 1, & g^1 \geq h^3 \\ \dfrac{h^3 - 2h^2 + 2g^2 - g^1}{2(h^3 - h^2 + g^2 - g^1)}, & g^2 > h^2, g^1 \leq h^3 \\ \dfrac{g^3 - h^1}{2(g^3 - g^2 + h^2 - h^1)}, & g^2 \leq h^2, g^3 > h^1 \\ 0, & g^3 \leq h^1 \end{cases} \tag{10b}$$

In Equations (9a) and (9b), by substituting $\tilde{g}$ with $\sum_{j=1}^{n} \widetilde{a_{ij}} x_j$, and by substituting $\tilde{h}$ with $\widetilde{b_U}$ and $\widetilde{b_L}$, respectively, Equations (9a) and (9b) are transformed into the left-hand side of the constraints in Equations (2b) and (2c), respectively. In addition, ζ_U and ζ_L on the right-hand side of the constraints Equations (2b) and (2c) should be greater than 0.5. As such, Equations (9a) and (9b) are limited to the constraints expressed by Equation (11) as follows:

$$Cr(\tilde{g} \leq \tilde{h}) = \begin{cases} \dfrac{w(g^3 - h^1) + h^2 - g^2}{g^3 - g^2 + h^2 - h^1}, & g^2 \leq h^2, g^3 > h^1 \\ \dfrac{w(h^3 - g^1)}{h^3 - h^2 + g^2 - g^1}, & g^2 > h^2, g^1 < h^3 \end{cases} \tag{11a}$$

$$Cr(\tilde{g} \geq \tilde{h}) = \begin{cases} \dfrac{w(h^3 - g^1) + g^2 - h^2}{h^3 - h^2 + g^2 - g^1 +}, & g^2 > h^2, g^1 \leq h^3 \\ \dfrac{w(g^3 - h^1)}{g^3 - g^2 + h^2 - h^1}, & g^2 \leq h^2, g^3 > h^1 \end{cases} \tag{11b}$$

The constraints expressed as Equations (2b) and (2c) become Equation (12), which is expressed as follows:

$$\begin{cases} \dfrac{w\left[\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3-(b_U)^1\right]+(b_U)^2-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2}{\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2+(b_U)^2-(b_U)^1}\geq\zeta_U, & \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2\leq(b_U)^2,\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3>(b_U)^1 \\ \dfrac{w\left[(b_U)^2-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1\right]}{(b_U)^3-(b_U)^2+\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1}\geq\zeta_U, & \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2>(b_U)^2,\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1<(b_U)^3 \end{cases} \tag{12a}$$

$$\begin{cases} \dfrac{w\left[((b_L)^3-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1\right]+\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2-(b_L)^2}{\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1+(b_L)^3-(b_L)^2}\geq\zeta_L, & \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2>(b_L)^2,\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1\leq(b_L)^3 \\ \dfrac{w\left[\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3-(b_L)^1\right]}{\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3-\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2+(b_L)^2-(b_L)^1}\geq\zeta_L, & \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2\leq(b_L)^2,\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3>(b_L)^1 \end{cases} \tag{12b}$$

The constraints expressed in Equation (12a) can be transformed into Equation (13), which is expressed as follows:

$$\begin{gathered}(\zeta_U-w)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3+(1-\zeta_U)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2\leq(1-\zeta_U)(b_U)^2+(\zeta_U-w)(b_U)^1 \\ \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2\leq(b_U)^2,\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3>(b_U)^1\end{gathered} \tag{13a}$$

$$\begin{gathered}(w-\zeta_U)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1+\zeta_U\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2\leq(w-\zeta_U)(b_U)^3+\zeta_U(b_U)^2 \\ \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2>(b_U)^2,\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1<(b_U)^3\end{gathered} \tag{13b}$$

Similarly, the constraints expressed in Equation (12b) can be transformed into Equation (14), which is expressed as follows:

$$(\zeta_L - w)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1 + (1-\zeta_L)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \geq (\zeta_L - w)(b_L)^3 + (1-\zeta_L)(b_L)^2$$
$$\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 > (b_L)^2,\ \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1 \leq (b_L)^3 \tag{14a}$$

$$(w-\zeta_L)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3 + \zeta_L\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \geq (w-\zeta_L)(b_L)^1 + \zeta_L(b_L)^2$$
$$\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \leq (b_L)^2,\ \left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3 > (b_L)^1 \tag{14b}$$

By combining Equations (13) and (14), four scenarios can be generated, which is the combination of Equations (13a) and (14a), Equation (13a) and Equation (14b), Equation (13b) and Equation (14a), and Equation (13b) and Equation (14b). However, the scenario of Equations (13b) and (14b) is impossible to obtain, since the value of $\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2$ cannot satisfy $\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 > (b_U)^2$ and $\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \leq (b_L)^2$ simultaneously. In addition, since the credibility levels for upper chlorine concentration limits ζ_U affect the solutions only when feasible solutions cannot be found, i.e., when feasible solutions can be obtained, the results are not affected by upper chlorine concentration limits ζ_U. As such, the solutions obtained through a combination of Equations (13a) and Equation (14a) and Equation (13b) and Equation (14a) are the same. Accordingly, only two scenarios are considered, which are the combination of Equations (13a) and (14a), termed scenario 1, and the combination of Equations (13a) and (14b), termed scenario 2, respectively.

When the weight coefficient w is equal to 1.0, Equation (12) can be transformed into Equation (15) and expressed as follows:

$$(1-\zeta_U)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1 + \zeta_U\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \leq (1-\zeta_U)(b_U)^3 + \zeta_U(b_U)^2 \tag{15a}$$

$$(1-\zeta_L)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3 + \zeta_L\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \geq (1-\zeta_L)(b_L)^1 + \zeta_L(b_L)^2 \tag{15b}$$

When the weight coefficient w is equal to 0.0, Equation (12) can be transformed into Equation (16) and expressed as follows:

$$(1-\zeta_U)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 + \zeta_U\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3 \leq (1-\zeta_U)(b_U)^2 + \zeta_U(b_U)^1 \tag{16a}$$

$$(1-\zeta_L)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 + \zeta_L\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1 \geq (1-\zeta_L)(b_L)^2 + \zeta_L(b_L)^3 \tag{16b}$$

When the weight coefficient w is equal to 0.5, Equation (12) can be transformed into Equation (17) and expressed as follows:

$$(2\zeta_U - 1)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^3 + 2(1-\zeta_U)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 \le 2(1-\zeta_U)(b_U)^2 + (2\zeta_U - 1)(b_U)^1 \tag{17a}$$

$$2(1-\zeta_L)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^2 + (2\zeta_L - 1)\left(\sum_{j=1}^{n}\widetilde{a_{ij}}x_j\right)^1 \ge (2\zeta_L - 1)(b_L)^3 + 2(1-\zeta_L)(b_L)^2 \tag{17b}$$

The FCCP model can generate more flexible solutions under various credibility levels and weight coefficients w and provide more information for decision managers.

2.2.2. Fuzzy credibility-constrained quadratic programming

With a combination of QP and fuzzy credibility-constrained programming (FCCP), a fuzzy credibility-constrained quadratic programming (FCCQP) model was proposed and expressed by Equation (18) as follows:

$$Minf = \sum_{j=1}^{n}[c_j + d_j(x_j)^2] \tag{18a}$$

subject to

$$Cr\left\{\sum_{j=1}^{n}\widetilde{a_{ij}}x_j \le \widetilde{b_U}\right\} \ge \zeta_U,\ \ i = 1, 2, \ldots, m \tag{18b}$$

$$Cr\left\{\sum_{j=1}^{n}\widetilde{a_{ij}}x_j \ge \widetilde{b_L}\right\} \ge \zeta_L, i = 1, 2, \ldots, m \tag{18c}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n \tag{18d}$$

According to the analysis above, the FCCQP model can be transformed into a crisp model and solved. The detailed process is summarized step by step as follows (also shown in Figure 2):

1. Formulate the FCCQP model with the objective function expressed as QP and uncertainties expressed as fuzzy sets;
2. Transform the FCCQP model into deterministic models according to the definition of credibility measures under various weight coefficients w through Equations (14)–(17);
3. Solve the deterministic models under a certain confidence level of upper bounds (ζ_U), lower bounds (ζ_L), and weight coefficient w to obtain feasible solutions;
4. Repeat steps 2 and 3 and obtain feasible solutions under various credibility levels of upper bounds (ζ_U) and lower bounds (ζ_L), and various weight coefficients w to obtain final solutions.

3. APPLICATION

3.1. Application of the FCCQP model to optimize chlorination scenarios under conditions of uncertainty

Since the boosters in the WDS should be designed with low operation and construction cost and the cost of boosters is a quadratic function, QP is required. In addition, the left-hand side and the right-hand side of the upper and lower limits constraints have fuzzy uncertainties, and, therefore, the FCCP model is required. By combining QP and FCCP, the FCCQP model is required to optimize the booster costs.

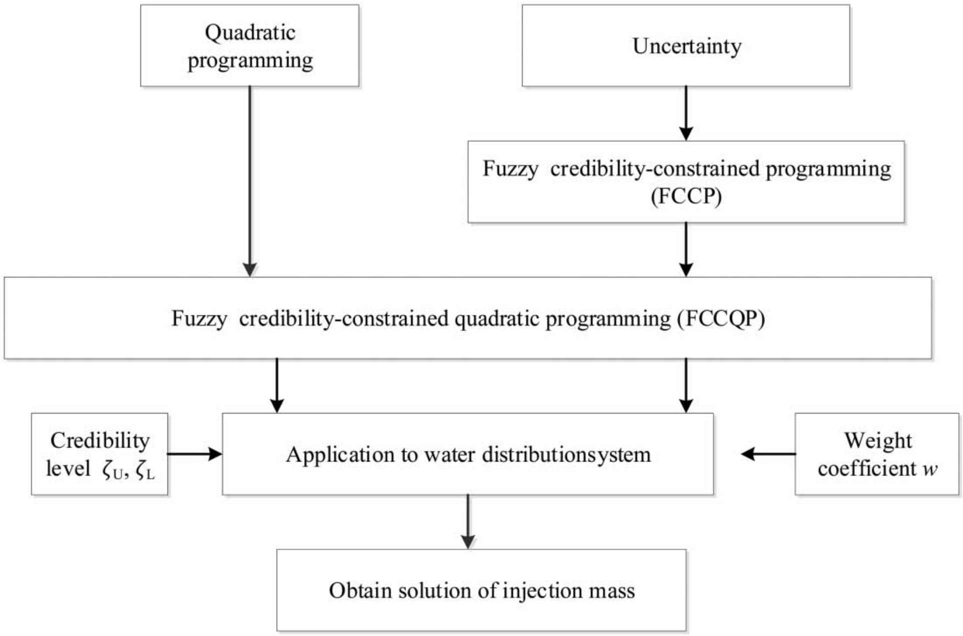


Figure 2 | The general framework of this study.

3.1.1. Objective function

The optimization problem is formulated by Equation (19) as follows:

$$Minf = \alpha \sum_{k=1}^{n_b} \sum_{l=1}^{n_t} x_{kl} \Delta t_l + \left(\sum_{k=1}^{n_b} \beta ((x_{kl})^{max})^{0.13} \right) \tag{19}$$

where f is the objective function including operation cost (OC) ($/day) and construction cost (CC) ($/day), x_{kl} are the decision variables representing the booster chlorination injection rate at time period l at booster station k (mg/min), Δt_l is the time duration for period l (min), n_b is the number of booster chlorination stations, n_t is the number of time periods, α and β are the coefficients in the cost function, which are assumed to be $2/mg and $2.21 (mg/min)$^{-0.13}$, respectively, and x_{kl}^{max} is the maximum booster injection rate (mg/min).

3.1.2. Constraints

The nodal residual chlorine concentration should be kept within acceptable limits, which are expressed by Equation (20) as follows:

$$Cr\left(\sum_{k=1}^{n_b} \sum_{l=1}^{n_t} (\widetilde{B_{ij}^{kl}}) x_{kl} \le \widetilde{C_U} \right) \ge \zeta_U, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{20a}$$

$$Cr\left(\sum_{k=1}^{n_b} \sum_{l=1}^{n_t} (\widetilde{B_{ij}^{kl}}) x_{kl} \ge \widetilde{C_L} \right) \ge \zeta_L, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{20b}$$

where $\widetilde{B_{ij}^{kl}}$ is the fuzzy response coefficients matrix of chlorine concentration at the node j at monitoring time i to the unit injection rate at booster or source location k at time l based on the superposition principle (Lansey *et al.* 2007), $\widetilde{C_U}$ and $\widetilde{C_L}$ are the fuzzy sets for acceptable upper and lower limits for chlorine concentration on the right-hand side of the constraints, ζ_U and ζ_L are predetermined confidence levels for constraints $Cr\left(\sum_{k=1}^{n_b} \sum_{l=1}^{n_t} (\widetilde{B_{ij}^{kl}}) x_{kl} \le \widetilde{C_U} \right)$ and $Cr\left(\sum_{k=1}^{n_b} \sum_{l=1}^{n_t} (\widetilde{B_{ij}^{kl}}) x_{kl} \ge \widetilde{C_L} \right)$, respectively.

Besides the constraints of chlorine concentration, the constraints of hydraulic balance, energy conservation, and non-negative conditions should also be satisfied, which can be found in the literature (Wang & Zhu 2021b). The fuzzy response coefficients matrix of chlorine concentration is obtained by hydraulic solver EPANET, where the hydraulic balance and energy conservation can be satisfied.

3.2. Example 1

The proposed methodology was applied to a small WDS (shown in Figure 3). Detailed information of the WDS can be found in the literature (Wang & Zhu 2021b). The lower bound, the most likely value, and the upper bound for fuzzy upper limit of chlorine concentration are set to be 3.5, 4.0, and 4.5 mg/L, respectively. Similarly, the lower bound, the most likely value, and the upper bound for fuzzy lower limit of chlorine concentration are set to be 0.15, 0.2, and 0.25 mg/L, respectively.

3.3. Example 2

In this paper, the Brushy Plain water distribution network system was applied, as shown in Figure 4. Detailed information of the WDS can also be found in the literature (Wang & Zhu 2021b). The most likely response coefficient matrix B_{ij}^{kl} was obtained by setting the source type as mass booster type with a time step of 1 h in a total of 24 h to be coincident with the hydraulic cycle time of 24 h. The lower bound and upper bound of B_{ij}^{kl} were set to be 0.9 and 1.1 times of the most likely response coefficient matrix. By simulating hydraulic and water quality analysis in 960 h to make sure that the system became stable and periodicity was obtained, the last 24 h analysis result was used. The global bulk and wall decay coefficients were set to be $k_b = 0.53$/day and $k_w = 5.1$ mm/day, respectively. The lower bound, the most likely value, and the upper bound for fuzzy upper and lower limits were the same as Example 1.

The FCCQP model can be solved by 'Solver' add-on in Microsoft Excel. By applying the FCCQP model, the total booster costs under various confidence level and weight coefficients were obtained.

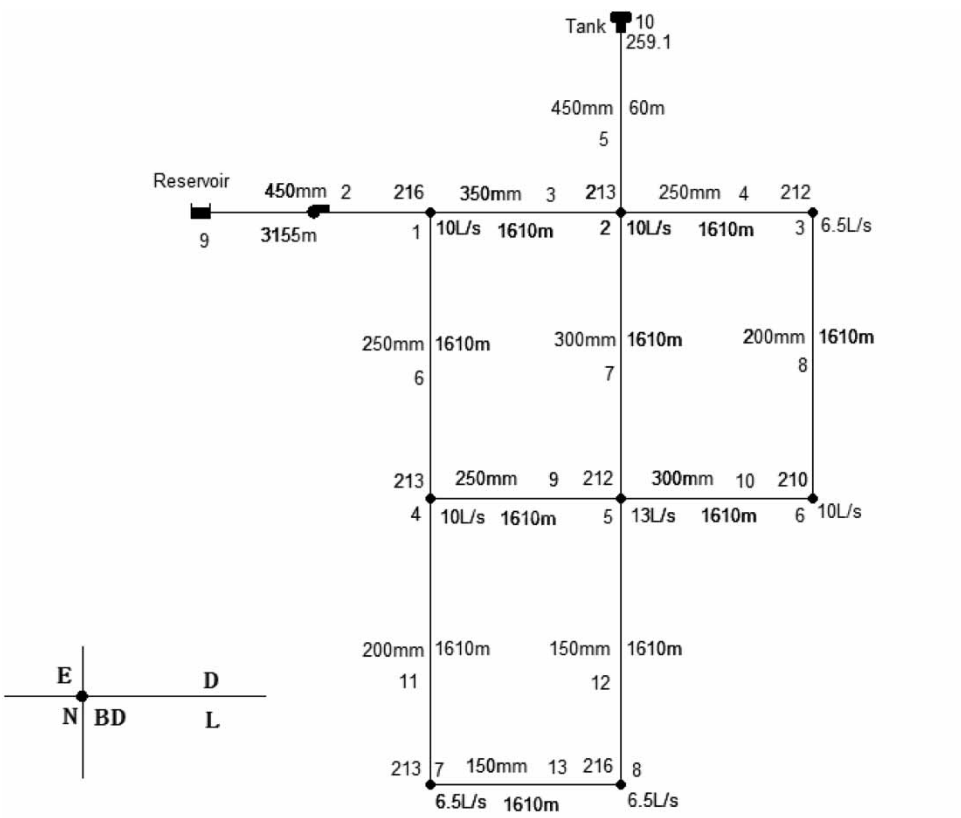


Figure 3 | Pipenet layout of Example 1.

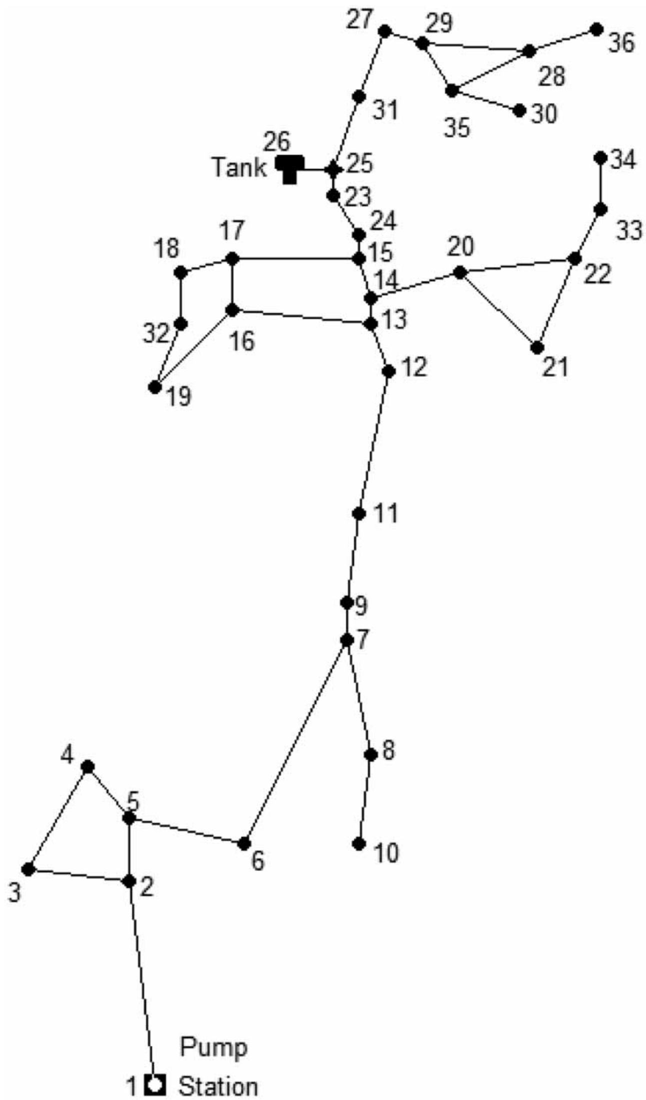


Figure 4 | Pipenet layout of Example 2.

4. RESULTS AND DISCUSSION

4.1. Application to example 1

In this study, the credibility levels ζ for upper and lower chlorine concentration limits were set to be 0.5, 0.6, 0.7, 0.8, and 0.9, respectively. The weight coefficients of probability w were set to be 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, respectively. The constraints connected with weight coefficients ranging from 0.1 to 0.9 were classified into scenario 1 and scenario 2. By solving the FCCQP model, the booster cost under various credibility levels ζ and weight coefficients w could be obtained, which could supply decision makers with sufficient information under uncertainty. The results obtained indicated that the booster costs had no relationship with the credibility levels for upper chlorine concentration limits. When $\zeta_U = 0.7$ and $\zeta_L = 0.5$, 0.6, 0.7, 0.8, and 0.9, the conditions were represented by Cases A–E, respectively. The booster costs under various cases and for weight coefficients w of 0.0, 0.5, and 1.0 are shown in Figure 5. When the weight coefficient of possibility w is 1.0, the booster costs for Cases A–E are \$26.37/day, \$27.13/day, \$27.90/day, \$28.69/day, and \$29.49/day, respectively. When the weight coefficient of possibility w is 0.0, the booster costs for Cases A–E are \$34.63/day, \$35.55/day, \$36.49/day, \$37.44/day, and \$38.42/day, respectively. When the weight coefficient of possibility w is 0.5, the booster costs for Cases A–E are \$30.31/day, \$31.99/day, \$33.73/day, \$35.55 /day, and \$37.44/day, respectively. These results indicated that the booster

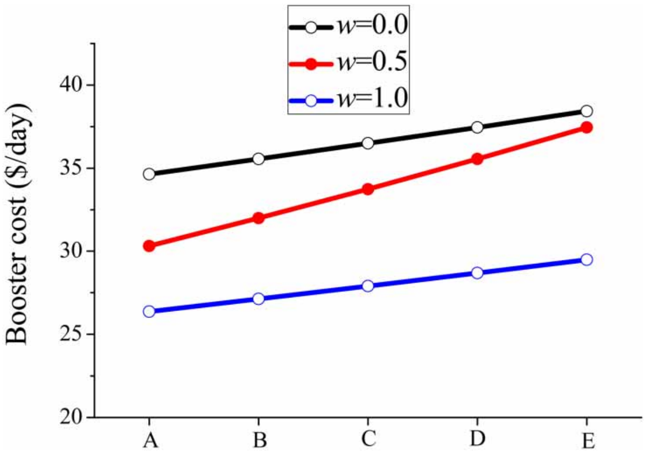


Figure 5 | Booster cost corresponding to $\zeta_U = 0.7$ and $\zeta_L = 0.5$ (a), 0.6 (b), 0.7 (c), 0.8 (d), and 0.9 (e) with weight coefficients w of 0.0, 0.5, and 1.0.

cost decreased when the weight coefficient of possibility increased from 0.0 to 1.0. In addition, the booster cost increased with the confidence level of lower chlorine concentration limits ζ_L.

In the booster costs, the proportions of operation cost and construction cost corresponding to Cases A–E are shown in Figure 6. The result indicated that the proportion of operation cost decreased with the weight coefficient w, which was the same with the variation in the total booster cost. However, the construction cost increased with the weight coefficient w. In addition, the proportion of operation cost increased with the confidence level of lower chlorine concentration limits ζ_L, and the proportion of construction cost decreased with the confidence level of lower chlorine concentration limits ζ_L. The results indicated that more booster injection mass was required with the increasing credibility levels for lower chlorine concentration limits ζ_L, which also led to an increase in booster cost and in the proportion of operation cost. The confidence level for lower chlorine concentration limits ζ_L refers to the degree of lower chlorine concentration limits constraint was satisfied, the results indicated that the WDS should be operated to inject more chlorine under more reliable level for lower chlorine concentration limits.

For the weight coefficients of w ranging from 0.0 to 1.0, two scenarios were considered (shown in Table 1). When the weight coefficient is 0.5, the booster cost for the two scenarios are the same values of \$30.31/day, \$31.99/day, \$33.73/day, \$35.55/day, and \$37.44/day corresponding to Cases A–E. The reason is that the constraints in the two scenarios become the same. In addition, the results obtained are of the same value of \$30.31/day under $w = 0.5$ for Case A, $w = 0.6$

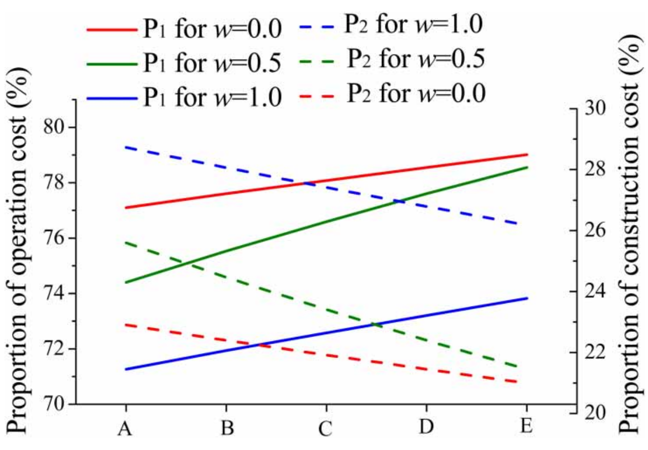


Figure 6 | Proportion of operation cost (P_1) and construction cost (P_2) corresponding to $\zeta_U = 0.7$ and $\zeta_L = 0.5$ (a), 0.6 (b), 0.7 (c), 0.8 (d), and 0.9 (e).

Table 1 | Booster cost for various cases, scenarios, and weight coefficients for Example 1

Booster cost ($/day)

Case		**A**		**B**		**C**		**D**		**E**	
Scenario		**S1**	**S2**	**S1**	**S2**	**S1**	**S2**	**S1**	**S2**	**S1**	**S2**
w	0.00	34.63		35.55		36.49		37.44		38.42	
	0.10	34.13	84.24	35.14	110.91	36.17	150.78	37.23	/	38.31	/
	0.20	33.51	44.74	34.63	50.70	35.78	57.43	36.96	65.11	38.17	73.95
	0.30	32.73	36.17	33.99	39.41	35.29	42.90	36.62	46.65	38.00	50.70
	0.40	31.70	32.42	33.14	34.63	34.63	36.96	36.17	39.41	37.77	42.00
	0.50	30.31		31.99		33.73		35.55		37.44	
	0.60	28.29	28.95	30.31	30.31	32.42	31.70	34.63	33.14	36.96	34.63
	0.70	25.13	28.01	27.64	29.14	30.31	30.31	33.14	31.50	36.17	32.73
	0.80	19.42	27.32	22.76	28.29	26.37	29.29	30.31	30.31	34.63	31.35
	0.90	0.00	26.79	10.60	27.64	16.32	28.51	22.76	29.40	30.31	30.31
	1.00	26.37		27.13		27.90		28.69		29.49	

Note: S1 refers to scenario 1 and S2 refers to scenario 2.

for Case B, $w = 0.7$ for Case C, $w = 0.8$ for Case D, and $w = 0.9$ for Case E, respectively. The reason is that the constraints expressed in Equations (14a) and (14b) are the same.

The comparisons of scenario 1 and scenario 2 for Cases–E are shown in Figure 7. For Case A, the two curves corresponding to scenario 1 and scenario 2 have only one intersection at a weight coefficient w value of 0.5 with the booster costs of $31.99/day, $33.73/day, $35.55/day, and $37.44/day (Figure 7(a). With the increasing confidence level for lower limits ζ_L, the booster costs increase from $31.99/day to $37.44/day. Besides the intersection of $w = 0.5$, booster costs at the other intersection are $30.31/day at a weight coefficient w value of 0.6 (Figure 7(b), 0.7 (Figure 7(c), 0.8 (Figure 7(d), and 0.9 (Figure 7(e) corresponding to Cases B–E. The results obtained can imply an interrelationship between the credibility levels of lower chlorine concentration limits ζ_L, the weight coefficients w, and booster costs. For scenarios 1 and 2, the booster cost decreases with the weight coefficients w. The booster cost is a concave function for scenario 1, and it is a convex function for scenario 2.

The FCCQP model can provide more information with various weight coefficients and can reflect the mutual interactions among possibility, necessity, and credibility measures. As such, the FCCQP model is more representative and generalized for optimizing the booster cost under conditions of uncertainty.

The comparisons of booster costs for Cases A–E in scenario 1 are shown in Figure 8. The booster cost increases with the confidence level of lower chlorine concentration limits ζ_L due to the booster cost increasing in the order of Case A < Case B < Case C < Case D < Case E, which is the same with the regulation shown in Figure 5. For example, when $w = 0.7$, the booster costs are $25.13/day, $27.64/day, $30.31/day, $33.14/day, and $36.17/day for Cases A–E, respectively. In addition, the increase magnitude increases when the weight coefficient value w increases from 0.0 to 0.9. However, when $w = 1.0$, the booster costs are $26.37/day, $27.13/day, $27.90/day, $28.69/day, and $29.49/day for Cases A–E, respectively. The increase magnitude is less than that under other w values of 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

In scenario 1, when the weight coefficient increases from 0.0 to 0.9, the booster cost decreases from $34.13/day to $0.00/day, from $35.14/day to $10.60/day, from $36.17/day to $16.32/day, from $37.23 /day to $22.76/day, and from $38.31/day to $30.31/day for Cases A–E, respectively. The decrease magnitude decreases in the order of Case A > Case B > Case C > Case D > Case E. When $w = 1.0$, there is a dramatic change in booster cost. For example, for Cases A–D, the booster costs at $w = 1.0$ are higher than the booster costs at $w = 0.9$, while for Case E, the booster cost at $w = 1.0$ is slightly lower than the booster costs at $w = 0.9$. The variations in weight coefficients refer to the change from possibility measure to necessity measure. Under the possibility measure, the booster costs are the greatest; however, under the necessity measure, the booster costs are not always the least for scenario 1.

The comparisons of booster cost for Cases A–E in scenario 2 are shown in Figure 9. Similar to scenario 1, the booster cost increases with the confidence level of lower chlorine concentration limits ζ_L due to the booster cost also increasing in the order of Case A < Case B < Case C < Case D < Case E, which is the same with the regulation shown in Figure 5. For example, when $w = 0.7$, the booster costs are $28.01/day, $29.14/day, $30.31/day, $31.50/day, and $32.73/day for Cases A–E,

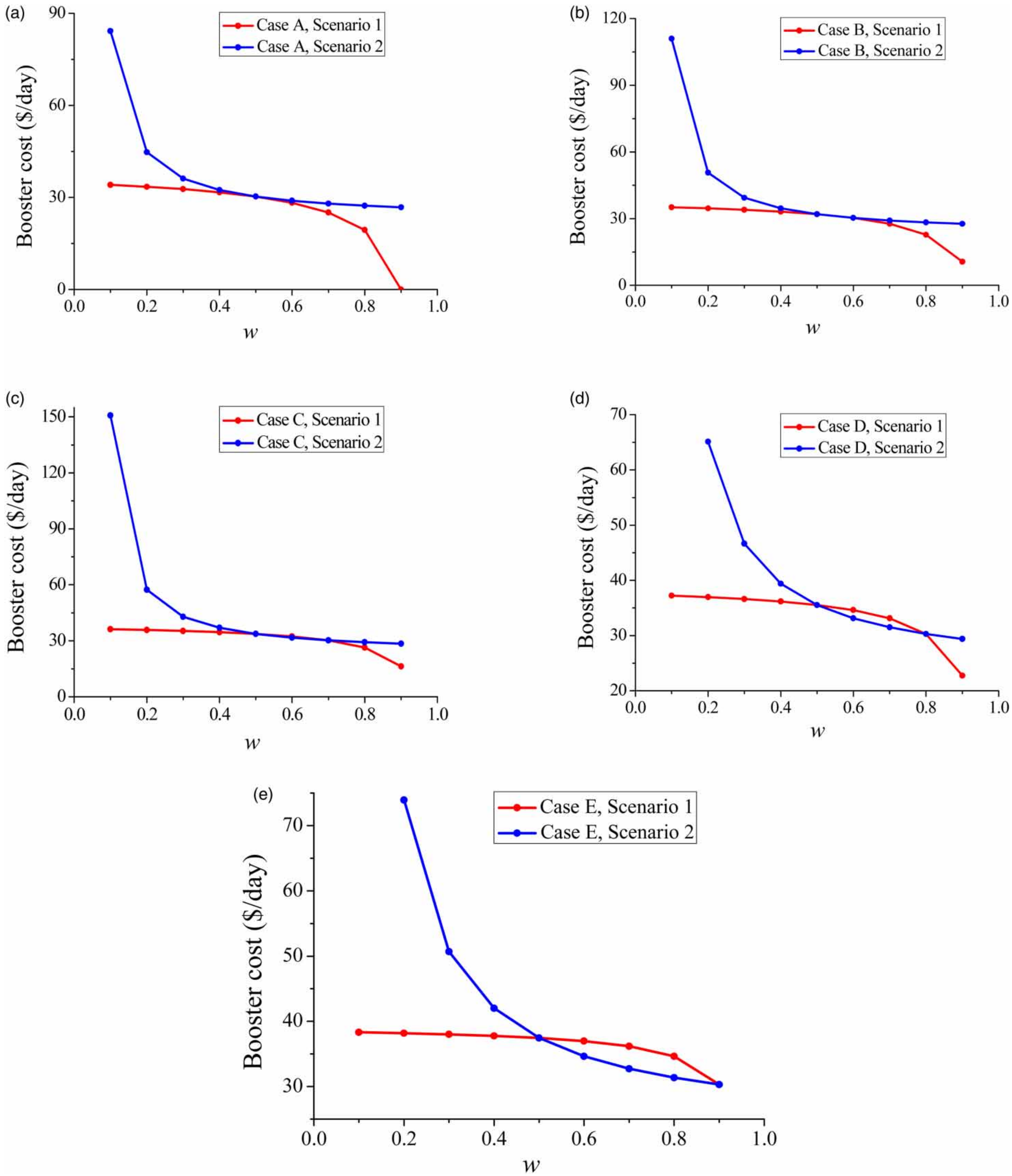


Figure 7 | Comparison of booster costs for scenario 1 and scenario 2 under various weight coefficients *w* and Case A (a), Case B (b), Case C (c), Case D (d), and Case E (e).

respectively. Different from scenario 1, the increase magnitude of booster cost is the greatest in case of $w = 0.1$, while in scenario1, the increase magnitude of booster cost is the greatest in the case of $w = 0.9$. In addition, when $w = 0.1$, feasible solutions cannot be obtained for Cases D and E due to the upper limit constraints.

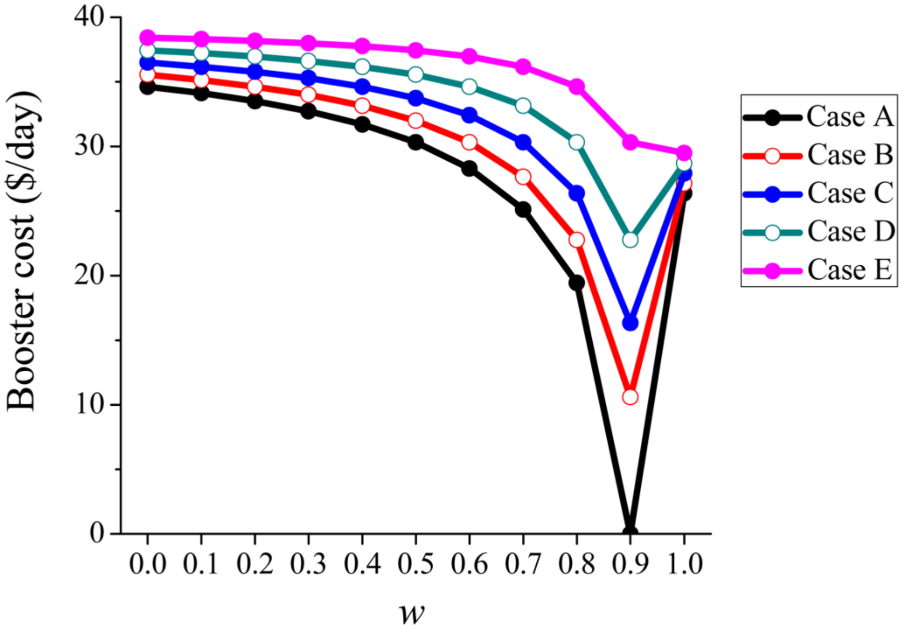


Figure 8 | Comparison of booster costs for Cases A–E under scenario 1.

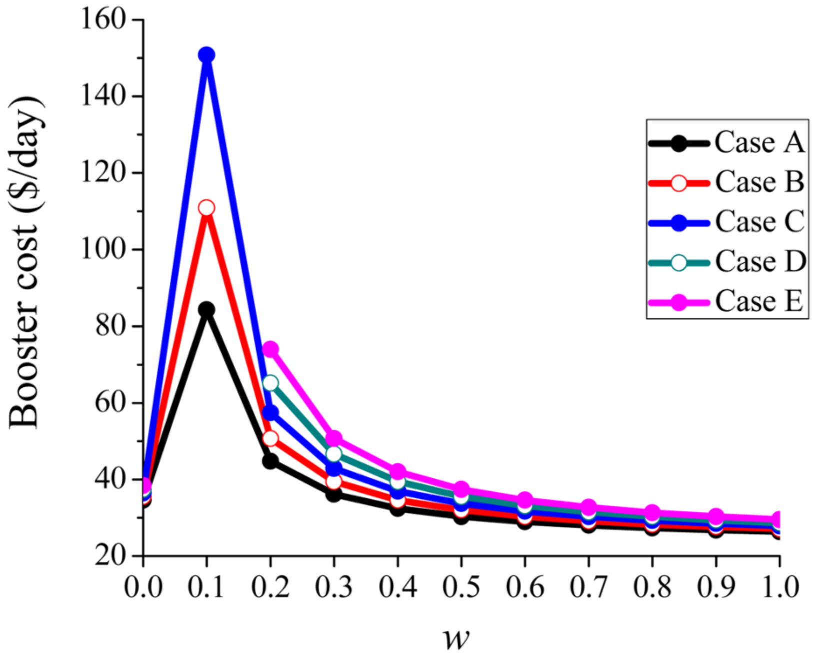


Figure 9 | Comparison of booster costs for Cases A–E under scenario 2.

In scenario 2, when the weight coefficient increases from 0.2 to 1.0, the booster cost decreases from $44.74/day to $26.37/day, from $50.70/day to $27.13/day, from $57.43 /day to $27.90/day, from $65.11/day to $28.69 /day, and from $73.95/day to $29.49/day for Cases A–E, respectively. Contrary to scenario 1, the decrease magnitude decreases in the order of Case E > Case D > Case C > Case B > Case A. The decrease magnitudes for Cases A–C in scenario 1 are larger than those in scenario 2, while for Cases D and E, the decrease magnitudes in scenario 1 are less than those in scenario 2. When $w = 0.0$, there is a dramatic change in booster cost. For example, for Cases A–E, the booster costs at $w = 0.0$ are \$34.63/day, \$35.55/day, \$36.49/day, \$37.44/day, and \$38.42/day, respectively, while booster costs at $w = 0.2$ for Cases A–E are \$44.74/day, \$50.70/day, \$57.43/day, \$65.11/day, and \$73.95/day. The booster costs at $w = 0.0$ are less than the booster cost at $w = 0.2$. In addition, the increase magnitude increases when the weight coefficient value w increases from 0.0 to 0.9. However, when $w = 1.0$, the booster costs are \$26.37/day, \$27.13/day, \$27.90 /day, \$28.69/day, and \$29.49/day for Cases A–E, respectively. The increase magnitude is less than that under other w values of 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

With reference to the same WDS with only fuzziness on the right-hand side of the constraints, the injection mass for Case C increases with the weight coefficients; however, in this paper with dual fuzzy uncertainties on both sides of the constraints, the injection mass for Case C decreases with the weight coefficients under two scenarios (shown in Figure 10). For weight coefficients lower than 0.7, the results obtained in scenario 1 are close to the results obtained in the reference, while for weight coefficients greater than 0.7, the results obtained in scenario 2 are close to the results obtained in the reference.

4.2. Application to Example 2

Similar to Example 1, the booster costs for various cases, scenarios, and weight coefficients for Example 2 are shown in Table 2. Cases A–E refer to $\zeta_U = 0.7$ and $\zeta_L = 0.5$, 0.6, 0.7, 0.8, and 0.9, respectively. When the weight coefficient is 0.5, the booster costs for the two scenarios are the same values of \$11.92/day, \$12.37/day, \$12.83/day, \$13.31/day, and \$13.81/day corresponding to Cases A–E. The reason is that the constraints in the two scenarios become the same. In addition, the results obtained are of the same value of \$11.92/day under $w = 0.6$ (Case B), $w = 0.7$ (Case C), $w = 0.8$ (Case D) and, $w = 0.9$ (Case E), respectively. The reason is that the constraints expressed in Equations (14a) and (14b) are the same. As such, there are two intersections between scenario 1 and scenario 2 at $w = 0.5$ and 0.6 for Case B, at $w = 0.5$ and 0.7 for Case C, at $w = 0.5$ and 0.8 for Case D, and at $w = 0.5$ and 0.9 for Case E, respectively.

The booster cost increases with the confidence level of lower chlorine concentration limits ζ_L due to the booster cost increasing in the order of Case A < Case B < Case C < Case D < Case E. For example, when $w = 0.7$ in scenario 1, the booster costs are \$10.52/day, \$11.20/day, \$11.92/day, \$12.68/day, and \$13.48/day for Cases A–E, respectively. When $w = 0.7$ in scenario 2, the booster costs are \$11.30/day, \$11.61/day, \$11.92/day, \$12.24/day, and \$12.57/day for Cases A–E, respectively. When the confidence level of lower chlorine concentration limits ζ_L is less than the value of w, the booster costs in scenario 1 are less than those in scenario 2. When the confidence level of lower chlorine concentration limits ζ_L is greater than the value of w, the booster costs in scenario 1 are larger than those in scenario 2. In scenario 2, a feasible solution cannot be found at $w = 0.1$ for Cases A–E, and at $w = 0.2$, a feasible solution also cannot be found for Case E. In addition, the increase magnitude increases when the weight coefficient value w increases from 0.0 to 0.9. However, when $w = 1.0$, the

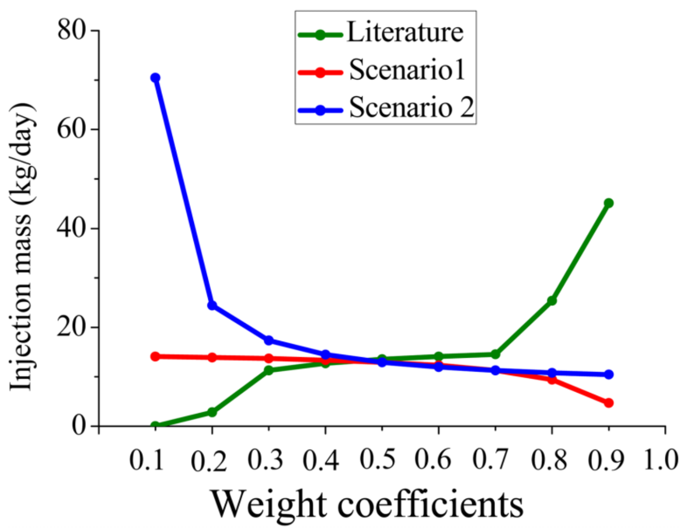


Figure 10 | Comparison of booster cost with the reference (Wang & Zhu 2021b).

Table 2 | Booster cost for various cases, scenarios, and weight coefficients for Example 2

Booster cost ($/day)											
Cases		**A**		**B**		**C**		**D**		**E**	
Scenarios		**S1**	**S2**	**S1**	**S2**	**S1**	**S2**	**S1**	**S2**	**S1**	**S2**
w	0.00	13.07		13.31		13.56		13.81		14.07	
	0.10	12.94	/	13.20	/	13.48	/	13.75	/	14.04	/
	0.20	12.77	15.71	13.07	17.25	13.37	18.98	13.68	20.94	14.00	/
	0.30	12.57	13.48	12.90	14.33	13.24	15.24	13.59	16.21	13.96	17.25
	0.40	12.29	12.48	12.68	13.07	13.07	13.68	13.48	14.33	13.90	15.00
	0.50	11.92		12.37		12.83		13.31		13.81	
	0.60	11.38	11.56	11.92	11.92	12.48	12.29	13.07	12.68	13.68	13.07
	0.70	10.52	11.30	11.20	11.61	11.92	11.92	12.68	12.24	13.48	12.57
	0.80	8.93	11.12	9.87	11.38	10.86	11.65	11.92	11.92	13.07	12.20
	0.90	0.00	10.97	6.26	11.20	8.04	11.44	9.87	11.68	11.92	11.92
	1.00	10.86		11.06		11.27		11.49		11.70	

Note: S1 refers to scenario 1, and S2 refers to scenario 2.

booster costs are $10.86/day, $11.06/day, $11.27/day, $11.49/day, and $11.70/day for Cases A–E, respectively. The increase magnitude is less than that under other w values of 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. In scenario 1, when the weight coefficient increases from 0.3 to 1.0, the booster cost decreases from $12.57/day to $0.00/day, from $12.90/day to $6.26/day, from $13.24/day to $8.04/day, from $13.59/day to $9.87/day, and from $13.96/day to $11.92/day for Cases A–E, respectively. In scenario 2, when the weight coefficient increases from 0.3 to 1.0, the booster cost decreases from $13.48/day to $10.97/day, from $14.33/day to $11.20/day, from $15.24/day to $11.44/day, from $16.21/day to $11.68/day, and from $17.25/day to $11.92/day for Cases A–E, respectively. Contrary to scenario 1, the decrease magnitude decreases in the order of Case E > Case D > Case C > Case B > Case A. Similar to Example 1, for Cases A–C the decrease magnitudes in scenario 1 are larger than those in scenario 2, while for Cases D and E, the decrease magnitudes in scenario 1 are less than those in scenario 2.

5. CONCLUSION

In this paper, a FCCQP model was developed for determining the optimization cost of booster in the WDS under fuzzy conditions of uncertainty, which could deal with fuzziness on both the left-hand and right-hand sides of constraints. The proposed model was applied to two examples to address the results of booster cost affected by credibility levels and weight coefficients.

The results indicated that the booster cost, as well as the proportion of operation cost, increases with the increase in the confidence level for lower chlorine concentration ζ_L, i.e., the booster cost and the proportion of operation cost increase in the order of Case A < Case B < Case C < Case D < Case E. In addition, the booster cost, as well as the proportion of operation cost, decreases with the weight coefficient w. In scenario 1, the decrease magnitudes decrease with the increase in the confidence level for lower chlorine concentration ζ_L, while in scenario 2, the decrease magnitudes increase with the increase in the confidence level for lower chlorine concentration ζ_L. The booster cost function curves with the variation of weight coefficients are concave and convex for scenario 1 and scenario 2, respectively. For Case A, there is one intersection for the two curves of scenarios 1 and 2, while for Cases B–E, there are two intersections for the two curves of scenarios 1 and 2. Compared with the reference with the fuzzy right-hand side constraint, the variation regulation is quite different for the two scenarios. The results obtained here can help managers to make informed decisions on disinfection injection under conditions of fuzzy uncertainty.

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ETHICS APPROVAL

Not applicable.

CONSENT TO PARTICIPATE

Not applicable.

CONSENT TO PUBLISH

Not applicable.

AUTHOR CONTRIBUTION

Y.W. conceptualized the study, prepared the methodology, and wrote, reviewed, and edited the article. G.Z. wrote, revised, and edited the article.

COMPETING INTERESTS

The authors declare no conflicts of interest/competing interests.

AVAILABILITY OF DATA AND MATERIALS

All relevant data are included in the paper or its Supplementary Information.

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